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A Non-Classical Model for Circular Mindlin Plates Incorporating Microstructure and Surface Energy Effects

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Abstract

A new non-classical model for circular Mindlin plates is developed using a modified couple stress theory, a surface elasticity theory and Hamilton's principle. The equations of motion and boundary conditions are simultaneously obtained through a variational formulation, and the microstructure and surface energy effects are treated in a unified manner. The new plate model contains a material length scale parameter to account for the microstructure effect and three surface elasticity constants to describe the surface energy effect. The current non-classical model includes the plate models considering the microstructure influence only and the surface energy effect alone as special cases, and it recovers the classical elasticity-based circular Mindlin plate model when both the microstructure and surface energy effects are suppressed. To illustrate the new model, the static bending problem of a clamped circular Mindlin plate under a uniform normal load is analytically solved by directly applying the general formulas derived.

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Keywords: Mindlin plate; size effect; couple stress theory; surface elasticity; Hamilton's principle; plate theory.

1. Introduction

Thin plates have been widely used in microelectromechanical systems and other industrial sectors. It has been experimentally observed that such plates exhibit microstructure-dependent size effects at the micron scale[1], which

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cannot be explained using classical elasticity due to the lack of any material length scale parameter. Hence, efforts have been made to develop non-classical plate models based on higher-order elasticity and surface elasticity theories.

Lazopoulos[2] provided a non-classical von Karman plate model based on a simplified strain gradient elasticity theory (SSGET)[3]. This SSGET was also employed by Papargyri-Beskou et al.[4] to derive non-classical governing equations for Kirchhoff plates. By using a constitutive relation in non-local elasticity suggested in Eringen[5], Lu et al.[6] proposed a Kirchhoff plate model and a Mindlin plate model without using a variational formulation. Reddy and Berry[7] studied axisymmetric bending of functionally graded circular plates employing a modified couple stress theory[8,9]. Recently, three models for Mindlin plates and third-order shear deformation plates have been developed by Ma et al.[10], Gao et al.[11] and Zhou and Gao[12] using the modified couple stress theory and Hamilton's principle.

The surface elasticity theory[13-17] has also been applied to develop non-classical models for thin plates involving surface energy effects. Lim and He[18] presented a geometrically nonlinear plate model for nano-scale films based on the Kirchhoff hypothesis and the von Karman strains. Lu et al.[19] constructed a size-dependent thin plate model by including the normal stress on and inside the surface of the bulk substrate. Wang and Wang[20] provided a model for non-linear free vibrations of a Kirchhoff plate and a Mindlin plate using the von Karman strains. Liu and Rajapakse[21] published a size-dependent continuum model for thin and thick circular plates.

However, very few models have been developed for thin plates by considering both the microstructure and surface energy effects. A non-classical Kirchhoff plate model, which is based on a modified couple stress theory and a surface elasticity theory, was presented in Shaat et al.[22] without using a variational formulation. Recently, non-classical models for Kirchhoff plates were developed by Zhang et al.[23] and Gao and Zhang[24] utilizing a variational formulation, a modified couple stress theory and a surface elasticity theory.

In the current paper, a non-classical model for circular Mindlin plates is provided by using the modified couple stress theory[8,9], the surface elasticity theory[13,14], and Hamilton's principle.

2. Formulation

Consider a flat thin circular plate of inner radius a, outer radius b and uniform thickness h, as shown in Fig. 1, where the cylindrical coordinate system (r, θ, z) is adopted, with the $r\theta$-plane being coincident with the geometrical mid-plane of the undeformed plate.

According to the Mindlin plate theory, the displacement field in a thin circular plate undergoing axisymmetric deformations can be written as[12,25]

$$u_r(r,\theta,z,t) = u(r,t) - z\phi_r(r,t),\;\; u_\theta(r,\theta,z,t) = 0,\;\; u_z(r,\theta,z,t) = w(r,t), \tag{1a-c}$$

where u_r, u_θ and u_z are, respectively, the radial, tangential and transverse components of the displacement vector $\mathbf{u}$ of a point (r, θ, z) in the plate at time t, u and w are, respectively, the radial and transverse components of the displacement vector of the corresponding point (r, θ, 0) on the plate mid-plane at time t, and ϕ_r is the rotation angle of a transverse normal line about the $\mathbf{e}_\theta$ direction (see Fig. 1).

In Fig. 1, S^+ and S^- denote, respectively, the upper and lower surface layers of the circular Mindlin plate. These two zero-thickness surface layers are perfectly bonded to the bulk plate material at $z = \pm\, h/2$ and have distinct properties from the bulk material. The bulk material satisfies the modified couple stress theory[8,9], while the surface layers are governed by the surface elasticity theory[13,14].

According to the modified couple stress theory[8,9],

$$\boldsymbol{\sigma} = \lambda\, \mathrm{tr}(\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon},\quad \mathbf{m} = 2\mu l^2\boldsymbol{\chi}, \tag{2a,b}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}[\nabla\mathbf{u} + (\nabla\mathbf{u})^T],\quad \boldsymbol{\chi} = \frac{1}{2}[\nabla\boldsymbol{\psi} + (\nabla\boldsymbol{\psi})^T], \tag{3a,b}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{m}$ is the deviatoric part of the couple stress tensor, $\mathbf{I}$ is the second-order identity tensor, λ and μ are the Lamé constants in classical elasticity, l is a material length scale parameter measuring the couple stress effect[26,27], $\boldsymbol{\varepsilon}$ is the infinitesimal strain tensor, $\boldsymbol{\chi}$ is the symmetric curvature tensor, ∇ denotes the gradient, the superscript T represents the transpose, $\mathbf{u}$ is the displacement vector, and $\boldsymbol{\psi}$ is the rotation vector defined by

$$\boldsymbol{\psi} = \frac{1}{2} curl\mathbf{u}\cdot \tag{4}$$