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State-Dependent Delay and Drill-String Dynamics

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Abstract

In this work, stability analysis carried out with a reduced-order model of a drill string is presented. This model, which is nonlinear in nature and contains a state-dependent delay, is used to study coupled axial and torsion dynamics of the system. The axial penetration rate and drill spin speed are used as control parameters, and the analysis reveals that the consideration of the state-dependent delay is critical for capturing the influence of the axial penetration rate. As a part of the analysis, the linearized system associated with state-dependent delay is constructed and the D-subdivision method is utilized. The results are presented in the form of stability charts in the plane of spin speed and cutting coefficient and in the plane of spin speed and penetration rate. These results confirm the earlier findings obtained through numerical means with full and reduced-order models.

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Keywords: Drill sting; State-dependent delay; Stability analysis; D-subdivision method

1. Introduction

Rotary drilling systems are used in oil and gas exploration operations. Long and slender flexible structures called drill strings are key components of these systems. These rotating, flexible structures often experience stick-slip oscillations (e.g., Liao, Balachandran, Karkoub, and Abdel-Magid[1]). As shown in earlier work (Richard, Germay, and Detournay[2,3]), the time-delay effect associated with bit-rock cutting mechanics can play an important role in causing the stick-slip behavior. Drill-string system models may be constructed through a variety of discretization means (e.g., references[1,4,5,6]). In earlier work[6,7] conducted in the authors' group, reduced-order models, finite-element based discretization, and the presence of the state-dependent delay have been discussed. In the present work, with the goal of further understanding the effect of this delay, the authors have used the reduced-order model presented in the earlier work of Besselink, Deonoel, and Nijmeijer[4] and Liu, Vlajic, Long, Meng, and Balachandran[7].

The remaining sections of this article are organized as follows. In Section 2, a reduced-order system with two degrees of freedom for describing the drill-string dynamics is presented. With the goal of reducing the number of parameters, the governing equations are rewritten in nondimensionalized form. Next, the linearization is carried out in Section 3 as a step towards stability analysis. In Section 4, the obtained linearized system is analyzed by using

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the D-subdivision method and the illustrative stability charts obtained are presented. Finally, concluding remarks are collected together and presented at the end.

2. Modeling and Nondimensionalization

In Figure 1, an illustrative model of a drill-string system is illustrated. The axial and torsion motions of interest are also shown. At the top end of the drill sting, the system is imposed with a constant axial speed V_0 and a rotation speed Ω_0. In terms of the axial displacement $Z(t)$ and the rotational displacement $\Phi(t)$, the governing equations take the form

$$\begin{aligned} M\ddot{Z}(t) + C_a\dot{Z}(t) + K_a(Z(t) - V_0 t) &= W_s - W_b(t) \\ I\ddot{\Phi}(t) + C_t\dot{\Phi}(t) + K_t(\Phi(t) - \Omega_0 t) &= -T_b(t) \end{aligned} \tag{1}$$

Here, M and I are the respective translational and rotational inertias, K_a and K_t are the respective translational stiffness and torsion stiffness, and C_a and C_t represent the respective translational damping and torsion damping. Furthermore, W_s is the sum of the weight of both the drill pipe and drill collar. W_b and T_b respectively denote the weight and torque on the bi, and they are both determined by bit-rock interactions. Each of them can be decomposed in terms of cutting and friction components, as follows.

$$\begin{aligned} W_b(t) &= W_{bc}(t) + W_{bf}(t) \\ T_b(t) &= T_{bc}(t) + T_{bf}(t) \end{aligned} \tag{2}$$

The subscripts bc and bf are used to denote the cutting and friction components, respectively. Following the earlier work of Detournay and Defourny[8], those components can be expressed as

$$\begin{aligned} W_{bc}(t) &= \epsilon a\zeta R(d(t))H(\dot{\Phi}(t)) \\ T_{bc}(t) &= \tfrac{1}{2}\epsilon a^2 R(d(t))H(\dot{\Phi}(t)) \\ W_{bf}(t) &= \sigma a l H(d(t))H(\dot{Z}(t)) \\ T_{bf}(t) &= \tfrac{1}{2}\mu\gamma a^2\sigma l sgn(\dot{\Phi})H(d(t))H(\dot{Z}(t)) \end{aligned} \tag{3}$$

where the $R(.)$ function is the unit ramp function and $H(.)$ is the Heaviside step function. The different parameters along with representative values are listed in Table 1.

A drag bit or polycrystalline diamond compact (PDC) bit is often used in drilling operations. This bit is assumed to have N identical blades that are symmetrically distributed. In Figure 2, two successive blades are shown along with the delayed states. For an individual blade, the instantaneous depth of cut can be determined as

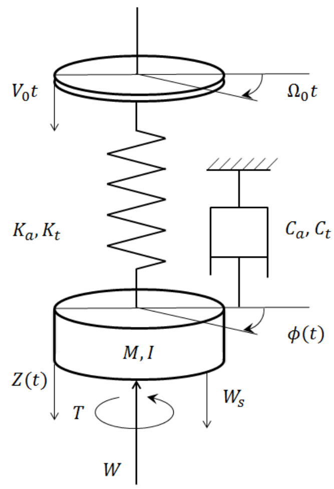

Fig. 1. Representative reduced-order model of drill-string system, following prior work[4,7].