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On the Coalescence of Spectral Values and its Effect on the Stability of Time-delay Systems: Application to Active Vibration Control

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Abstract

For linear delay-differential equations, a question of ongoing interest is to determine conditions on the equation parameters that guarantee exponential stability of solutions. Recent results have shown an unexpected link between the stable manifold and the variety characterizing multiple characteristic spectral values allowing to the right-most root assignment. In this paper, such an idea is presented and exploited in the control of active vibrations.

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Keywords: Retarded functional differential equations, asymptotic stability, active vibration control, rightmost root, root assignment

1. Introduction

To the best of the authors' knowledge, the theoretical use of time-delay in controllers design were first introduced in[1] where it is shown that the conventional proportional controller equipped with an appropriate time-delay performs an averaged derivative action and thus can replace the proportional-derivative controller. The interest of considering such delayed control laws of lies in the simplicity of the controller as well as in its easy practical implementation. In particular, it is proven in[2] that a chain of n integrators can be stabilized using n distinct delay blocks, where a delay block is described by two parameters: "gain" and "delay", see also[3]. In the context of mechanical engineering problems, the effect of time-delay was emphasized in[4] where concrete applications are studied, such as, for instance, the machine tool vibrations and robotic systems[5], see also[6] where the stabilization of an inverted pendulum is considered.

In recent works, a new interesting property of time-delay systems was emphasized. As a matter of fact, the multiple spectral values for time-delay systems was characterized by using a Birkhoff/Vandermonde-based approach, (see for instance[7,8,9,10]). More precisely, in[8], it is shown that the admissible multiplicity of the zero spectral value is bounded by the generic *Pólya and Szegő bound* denoted PS_B, which is nothing else than the *degree* of the corresponding quasipolynomial[11]. In[7], it is shown that a given *Crossing Imaginary Root* (CIR) with non vanishing frequency never

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reaches PS_B and a sharper bound for its admissible multiplicities is established. Furthermore, in [10], it is shown that the variety corresponding to a multiple root defines a stable variety for the steady state. Furthermore, it is emphasized that such a multiple spectral value corresponds to the spectral abscissa, see also [12,13]. Finally, in [14], the use of the above cited results allows constructing a design approach of delayed-feedback control law based on the rightmost-root assignment. In this work, we would like to experiment the applicability of the methodology described in [14] in damping one mode of vibrations for a piezo-actuated beam, modelled by Finite Element method, and used in [15].

2. System description

The problem of active vibration damping of thin mechanical structures is a topic that has received a great attention by the control community since several years [16], especially when actuators and sensors are based on piezoelectric materials. For mechanical structures that are deformable, piezoelectric materials are used as strain sensors or strain actuators. With an appropriate controller, they allow to achieve shape control [17,18] or the active damping of multi-modal vibrations thanks to their very large bandwidth. Moreover, their behavior is quite linear when they work in a specific range of use. This explains in part the great interest of using piezoelectric materials for the instrumentation of thin mechanical structures. In this area, the major challenge is the design of controllers able to damp the most vibrating modes in a specified low-frequency bandwidth while ensuring robustness against high-frequency modes, outside the bandwidth of interest, often unmodelled or weakly modelled. The inherent feature of this kind of systems is that they arise robustness issues when they are tackled with finite dimensional control tools. Many works have concerned the vibration control problem of the "*Euler-Bernoulli beam*" equipped with one rectangular piezoelectric actuator and sometime, another one, identical and collocated, but used as sensor. See for example [19], [20] where one edge of the beam is clamped whereas the other remains free. Other works dealt with the problem of vibration control for laminated rectangular plates [21] or complex plate like structures [22].

In this work, we consider the flexible structure depicted in Fig. 1. It is an aluminium-based beam, embedded in a mobile support. The mobile support is subjected to an acceleration, denoted by w in the sequel, and it is moving along the z axis. This flexible beam is equipped with two piezoelectric patches made with *lead zirconate titanate (also called PZT)*. One of them is used as an actuator and the other works as a sensor. These patches are supposed to be rigidly bounded on the beam, one on each side, located at the clamped edge. The whole device is called thereafter as a *piezo-actuated beam*. It can be deformed by the application of a voltage, denoted by u, across the actuator. The sensor delivers an electrical voltage which corresponds to a measure, denoted by y, of the local deformation under the piezoelectric patch. Very often, this equipped mechanical structure is partly described by the in-plane

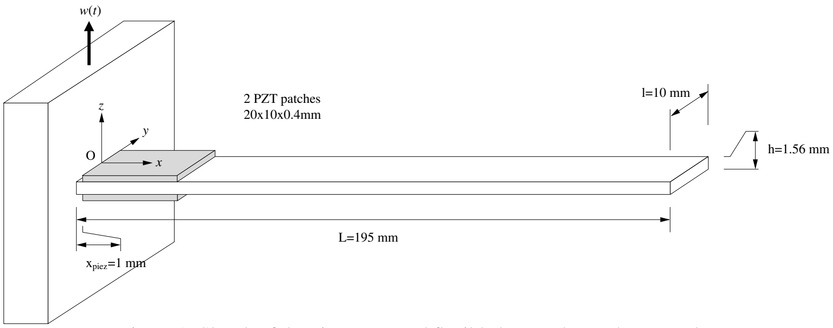

Figure 1. Sketch of the piezo-actuated flexible beam, clamped at one edge.

Euler-Bernoulli Partial Differential Equation (PDE) that suffers from the lack of precision in describing the electro-mechanical interactions between the passive structure and the piezoelectric components. Indeed, these latter are often withdrawn in the computation of the eigenfrequencies [23] of the whole structure. Nevertheless, such a structure obeys to fundamental equations of continuum mechanics in 3D space [24], involving computations of gradient of displacement vector and divergence operator applied to strength tensor. When completed with Neumann and Dirichlet boundary conditions, the fundamental equations give several PDEs that are coupled, thus that are hardly or impossible to solve analytically. Then, for controller design purposes, one naturally turns toward numerical methods in order to get the inputs-to-outputs dynamical models [25].