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Design of a delay-based controller for fast stabilization in a network system with input delays via the Lambert W function☆

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Abstract

Study of networks with multiple agent dynamics has been a central focus in systems and control community, with numerous results. Among them is the study of how time delays in sensing and actuation could affect network stability, and/or control design for the agents. However, little is known in published work regarding how to design agents' protocols analytically to achieve fast stability reaching. Here, an approach is developed to optimize the rightmost roots of a class of large scale LTI network dynamics via Lambert W function, in order to accomplish this. Results indicate that a delay explicitly present in the controller as a coefficient can create conditions for arbitrary rightmost root assignment. This feature is scalable, can be implemented with ease, and applied effectively for relatively large/small delays.

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Keywords: Time-delay systems, rightmost root analysis, pole placement, delay-based control, Lambert W function.

1. Introduction

Study of multi-agent systems has attracted tremendous attention especially in the past decade [1,2], with applications involving robotic networks [3], traffic flow dynamics [4], and collaborative human-robot systems [5]. While such systems can enjoy rich information flow amongst the agents with the network interconnectivity, distributed nature of the agents and the need to utilize advanced technologies to tailor these agents inevitably bring about a number of unique challenges to the design and control of multi-agent systems. One key challenge is the presence of time delays in the network dynamics [6,7,2], which may arise due to a number of reasons including agents actuation times, the need to use a communication medium to enable the agents to exchange information, and necessary computation times to process and infer large stream of data.

The presence of time delay in a dynamical system often imports undesirable characteristics, including poor performance, oscillatory response, and instability. Nevertheless, if carefully engineered, time delay can also be used as a

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vehicle to craft the dynamic response[8,9,10]. On the other hand, in the context of multi-agent systems, majority of published work has been focused on stability and control design, and few studies focused on achieving fast stabilization. Achieving fast stabilization even in the case of linear time-invariant (LTI) multi-agent systems is not straightforward since the arising dynamics due to delays is infinite dimensional making standard pole placement techniques inapplicable. While certain control approaches can be implemented to improve dynamic performance[11,12], rigorous control design tools to analytically and systematically design such controllers especially for large scale problems still do not exist in the literature.

One opportunity is to utilize reliable computational tools to approximate the rightmost eigenvalues of the dynamics[13], or to use such tools to tune the controller gains via optimization schemes[14]. On the other hand, effective design of rightmost eigenvalues depends on sufficient separation in the spectrum between these eigenvalues and those that are non-dominant. This was the focus of recent studies, where a measure of how well the spectrum separates is calculated to evaluate the efficacy of the pole placement efforts[15]. To this end, it is worthy to cite analytical techniques to perform pole placement, based on the use of resultants and discriminants[16,17,18]. Other ideas to achieve fast stabilization include strategically removing certain links between some of the agents to expedite consensus reaching[19], or re-designing the coupling strengths of the agents[20].

To the best knowledge of the authors, pole placement for LTI multi-agent network systems based on an analytical approach has so far not been studied. For a class of LTI multi-agent dynamics, here we propose a delay-based controller to select agents' couplings, which then enables a practical decomposition of the corresponding characteristic equation of the multi-agent system into subsystems. With these subsystems being in a particular form, Lambert W function technique is used next to tune the controller parameters without any approximation, which ultimately shifts the spectrum of the subsystems all at once, thereby yielding fast stabilization. The technique is scalable, easy to implement, and can be utilized even for relatively large delays.

For clarity purposes, the article primarily focuses on a special class of LTI systems, namely, a widely studied consensus dynamics, solely for the objective of fast stabilization. The dynamics is introduced in Section 2, along with the Lambert W function and the delay margin concepts. In Section 3, the main ideas of the proposed delay-based controller are presented on this dynamics using the Lambert W function. Here, non-dimensionalization is also performed resulting in simple algebraic tuning formulae for the parameters of the proposed controller, and an outline is provided laying out how to extend the approach to problems beyond the special case of consensus dynamics. Section 4 presents concluding remarks and further directions on research.

2. Preliminaries and problem statement

We first describe the consensus dynamics, its inherent properties, and the delay margin concept. Next, a brief description follows regarding the Lambert W function and its use for the analysis of Linear Time-Invariant Time-Delays Systems (LTI-TDS). These concepts are summarized from[6,21]. Then, the stability properties of the consensus dynamics are formulated in terms of the Lambert W function and associated to the delay margin of the system at hand.

2.1. Consensus Dynamics and its Delay Margin

Consider the following consensus dynamics with input delay $\tau > 0$,

$$\dot{x}_i(t) = \sum_{j=1, j\neq i}^{n} a_{ij}[x_j(t-\tau) - x_i(t-\tau)], \quad i = \overline{1,n}, \tag{1}$$

where x_i is the state of agent i, and the constant $a_{ij} > 0$ is the coupling strength between agents i and j. This dynamics and a number of its variations have been broadly studied in the literature in the context of neural networks[22], synchronization[23], traffic flow[7,4] and autonomous agents[2,12].

For convenience, we rewrite (1) in matrix form as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t-\tau), \tag{2}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the solution of system (2) at the time $t \geq 0$, and $\mathbf{A} = \{a_{ij}\}$ is a real $n \times n$ matrix, also known as the configuration matrix. Assuming that agents are connected, matrix $\mathbf{A}$ has a zero eigenvalue $\lambda_1 = 0$ corresponding to