Available online at www.sciencedirect.com

ScienceDirect

Procedia IUTAM 22 (2017) 99 – 106

Procedia IUTAM

www.elsevier.com/locate/procedia

IUTAM Symposium on Nonlinear and Delayed Dynamics of Mechatronic Systems

Initiation and Directional Control of Period-1 Rotation for Vertically or Horizontally Excited Parametric Pendulum

Santanu Das[a], Pankaj Wahi[a,*]

[a]*Department of Mechanical Engineering, Indian Institute of Technology Kanpur, Kanpur, UP-208016, India*

Abstract

Initiation of period-1 rotation of a vertically or horizontally excited parametric pendulum from arbitrary initial conditions has been studied using a switch delayed feedback control. We also study the possibility of controlling the preferential direction of rotation. In order to choose the control parameters, a systematic linear stability analysis of the various periodic solutions is performed using the Floquet theory. The control gains are obtained using the variation of the dominant Floquet multipliers with the control gain. Our procedure has been verified to work well with both the case of vertical and horizontal excitation.

Peer-review under responsibility of organizing committee of the IUTAM Symposium on Nonlinear and Delayed Dynamics of Mechatronic Systems

Keywords: Parametric pendulum; Energy harvesting; Period-1 rotation; Basin of attraction; Delayed feedback control; Directional control.

1. Introduction

Global warming and pollution caused by fossil fuels has led to a growing interest towards alternative energy sources. Wiercigroch[1] has proposed rotating motions of a pendulum floating on the sea waves as one such alternative source to run a turbine and generate electricity. From Fig. 1, it is clear that the proposed energy extractor of Wiercigroch[1] takes the form of a base excited (vertically) pendulum. Though this energy extractor takes the form of vertically excited parametric pendulum, we will talk about a vertically and a horizontally excited parametric pendulum since it is possible to convert vertical rectilinear motions into horizontal ones using some mechanisms. A base-excited damped parametric pendulum exhibits various kinds of steady state solutions and rotation is one of them[2]. This rotating solution can be used to extract energy from sea waves. Experiments have been done to confirm the practical feasibility of extracting energy from water waves[3,4]. However, for a fixed set of parameters, rotation is not the only solution that exists for a base excited damped parametric pendulum. So the basin of attraction of rotating solution does not encompass the whole initial condition space, i.e., $(\theta(0), \dot{\theta}(0))$ space. Furthermore Lenci and Rega[3] showed using the concept of dynamical integrity that experimental basin of attraction for rotation is smaller than the theoretical one. For these reasons, a control torque is needed to initiate rotation from all initial conditions. The period-1 rotation of base excited parametric pendulum exists for a large range of parameters among all the rotating solutions. Also

* Pankaj Wahi. Tel.: +91-512-259-6092 ; fax: +91-512-259-7408.
E-mail address: wahi@iitk.ac.in

2210-9838
Peer-review under responsibility of organizing committee of the IUTAM Symposium on Nonlinear and Delayed Dynamics of Mechatronic Systems
doi:10.1016/j.piutam.2017.08.013

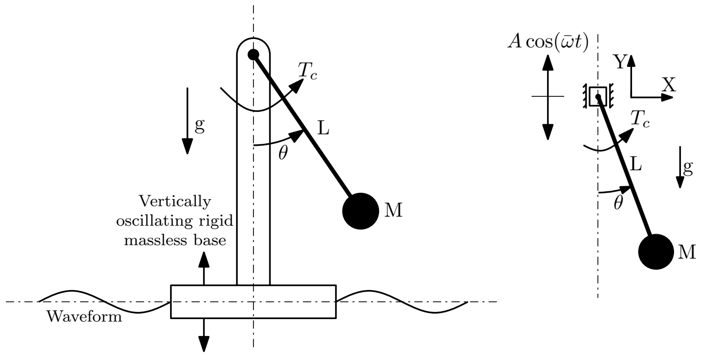

Fig. 1: Left: A schematic of working principle for energy extraction. Right: Simplified physical model[9].

period-1 rotation exists for small values of amplitude of excitation[2]. Hence, in this paper, we discuss the initiation and directional control of period-1 rotation for both these cases of parametric excitation separately.

The control law for initiation can not be chosen arbitrarily. It should not destabilize the period-1 rotation and should do no work for a period-1 rotation so that energy can be extracted from sea waves. Some researchers have proposed such kind of control laws[5,6] using extensions of the delayed control of Pyragas[7]. The control proposed by Yokoi and Hikihara[5] is smooth in nature and it initiates a preferential direction of rotation from all initial conditions. But it does not work for all sets of parameters. The control proposed by Najdecka[6] initiates both anti-clockwise and clockwise rotation, but cannot initiate a preferential direction of rotation. In this paper, we first propose a new switch delayed control which initiates both anti-clockwise and clockwise rotation from all initial conditions. Then we introduce a small modification to this control law to initiate a preferential direction of rotation.

2. Need for an External Control Torque

The physical model shown in Fig. 1 is nothing but a vertically excited pendulum with a torsional damper (Damping coefficient = C) at the pivot which signifies energy extraction and an external torque which represents the controller actuator. Using the time scale corresponding to the excitation frequency $\bar{\omega}$, we get the non-dimensionalized EOM as

$$\ddot{\theta} + c\dot{\theta} + (a - b\cos t)\sin\theta = F_c, \tag{1}$$

where $c = \frac{C}{\bar{\omega}ML^2}$, $a = \left(\frac{\omega_n}{\bar{\omega}}\right)^2$, $b = \frac{A}{L}$ and $F_c = \frac{T_c}{\bar{\omega}^2 ML^2}$. Similarly the non-dimensionalized EOM for a horizontally excited damped parametric pendulum with an external torque is

$$\ddot{\theta} + c\dot{\theta} + a\sin\theta - b\cos t\cos\theta = F_c\,. \tag{2}$$

The need for external control torque can be understood by studying the uncontrolled dynamics of the respective systems. For $F_c = 0$, Eqs. (1) and (2) reduce to

$$\ddot{\theta} + c\dot{\theta} + (a - b\cos t)\sin\theta = 0\,, \quad \text{and} \tag{3}$$

$$\ddot{\theta} + c\dot{\theta} + a\sin\theta - b\cos t\cos\theta = 0\,, \tag{4}$$

respectively. Rotation always exists in pair (anti-clockwise and clockwise) for a vertically and an horizontally excited pendulum. Equation (3) is symmetric, so that both θ and $-\theta$ are solutions. In case of Eq. (4), $\theta(t)$ and $-\theta(t-\pi)$ form the correct pair of solutions. However, rotary solutions are not the only attractors for any fixed set of parameters in case of vertically excited pendulum[2,8] and it also holds for the horizontally excited pendulum. As an example, for $a = 0.5$, $b = 0.1$ and $c = 0.03$, the co-exiting attractors of Eq. (3) are:

- The fixed point $(\theta, \dot{\theta}) = (0, 0)$,