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# Nonlinear waves in a two-ring network with a transverse coupling

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**Abstract**

A two-ring network coupling by a transverse link is considered in this paper. Divide and conquer algorithm is used to analyze the stability and Hopf bifurcation values. Nonlinear waves (such as synchronization and reflection waves) are studied using the center manifold theorem and normal form theory. It is shown that appropriate settings of transverse coupling can guarantee the backup of the complicated dynamics from one ring to the other.

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*Keywords:* Complex network; divide and conquer algorithm; nonlinear waves; Hopf bifurcation; spatio-temporal dynamics.

## 1. Introduction

The studies of theory and application of network science have currently pervaded various fields of science and engineering. Undoubtedly, many systems in nature can be described by models of networks, which are structures consisting of nodes or vertices connected by links or edges [1-4]. The ubiquity of complex networks in science and technology has naturally led to a set of common and important research problems concerning how the network structure facilitates and affects the dynamical behaviors of systems. What is the most efficient and robust architecture for the system following the changing of environment?

Rings networks are one of the classical regular networks, and are studied to gain insight into the mechanisms underlying the behavior of systems [5]. Rings networks are used in the biological world to model such structures as the cross-section through a plant stem. In the world of physics, they have been used to model rings of semiconductors lasers. A computer network configuration where the devices are connected to each other in a circular shape is another example for the ring network [6].

Ring topology provides only one pathway between any two nodes; a node failure or link break might isolate every node attached to the ring. Thus, it is natural to consider more complex networks to avoid the weakness of a ring topology. In response, some ring networks add a "counter-rotating ring" (C-Ring) to form a redundant topology: in the event of a break, data are wrapped back onto the complementary ring before reaching the end of the cable,

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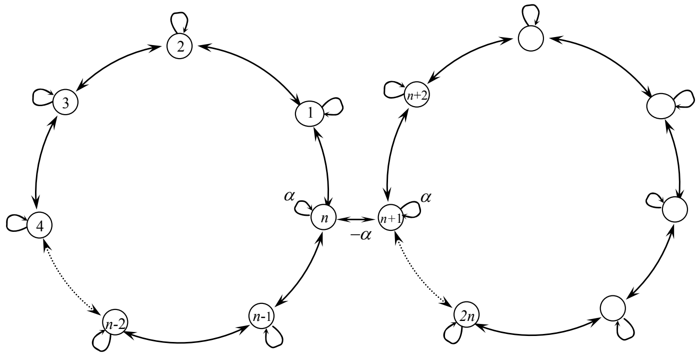


Fig. 1. Schematic of the two rings coupled transversally by a different type of coupling.

maintaining a path to every node along the resulting C-Ring. Another possible extension of the ring network consists of two-ring system, where the nodes within each ring have identical coupling; while the different transverse couplings are added between the two rings. There are two rings, active path and backup path. The information can be transferred from one ring to the other. In normal operations, traffic on the backup path is either blocked or ignored. If there is a failure in any of the network nodes or a link-loss, it will automatically redirect the disrupted traffic to the backup. After the affected path is repaired, the network will again be reconfigured operational status. This topology is an effective solution to meeting the requirements for link-loss backup in industrial normal operations.

In this paper, we consider a two-ring network coupling by a transverse link. The system is described by

$$\dot{x}_j = \begin{cases} \kappa x_j + F(x_j, \bar{x}_j) + \varepsilon[2x_j - x_{j-1} - x_{j+1}] & j \neq n, n+1 \\ \kappa x_j + F(x_j, \bar{x}_j) + \varepsilon[2x_j - x_{j-1} - x_1 + \alpha(x_j - x_{j+1})] & j = n \\ \kappa x_j + F(x_j, \bar{x}_j) + \varepsilon[2x_j - x_{j+1} - x_{2n} + \alpha(x_j - x_{j-1})] & j = n+1 \end{cases} \tag{1}$$

where $x_j \in \mathbb{C}, \kappa = \gamma + \mathrm{i}\Omega, \varepsilon = \varepsilon_r + \mathrm{i}\varepsilon_q$ with $\varepsilon_r, \varepsilon_q \in \mathbb{R}, \mathrm{i}^2 = -1, j = 1, 2, ..., 2n$. $F(.)$ is the nonlinear terms of an uncoupled oscillator. This paper assumes $\varepsilon_r < 0$ for simplicity.

In Eq. (1), connection strength between $n$ and $n+1$ is assumed to be $-\alpha$, the others are -1. Self-feedback strength of nodes $n$ and $n+1$ are $2+\alpha$, and the others are 2. The main reason for this setting is to ensure the conservation of the system. The schematic of the system is shown in Fig. 1.

In the present work, we shall study not only the stability but also the periodic local dynamics. The study will concern mainly the interaction of complicated behaviors between the two rings which leads to richer spatio-temporal dynamics, such as mirror-reflecting waves, resonance wave, et al. And we also want to know if the dynamics in one ring can be the backup of those in the other ring.

## 2. Presentations of the system

Eq. (1) can be written as the matrix form

$$\dot{\mathbf{x}} = (\gamma + \mathrm{i}\Omega)\mathbf{x} + (\varepsilon_r + \mathrm{i}\varepsilon_q)\mathbf{A}\mathbf{x} + \mathbf{F}, \tag{2}$$

where $\mathbf{F} = [F(x_1, \bar{x}_1), F(x_2, \bar{x}_2), ..., F(x_{2n}, \bar{x}_{2n})]^{\mathrm{T}}$, $\mathbf{A} = \mathrm{diag}(\mathbf{A_n}, \mathbf{A_n}) + \alpha \mathrm{R}$, with

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & ... & -1 \\ -1 & 2 & -1 & ... \\ ... & ... & ... & ... \\ -1 & ... & -1 & 2 \end{bmatrix}, \quad \mathrm{R} = \begin{bmatrix} \ddots & & & \ddots \\ & 1 & -1 & \\ & -1 & 1 & \\ \ddots & & & \ddots \end{bmatrix} \tag{3}$$

Note the rank of matrix R is one, and thus it can be written as $\mathrm{R} = \mathbf{v}\mathbf{v}^{\mathrm{H}}$, where $\mathbf{v}^{\mathrm{H}}$ denotes the conjugate transpose of the $2n$-order vector $\mathbf{v}$: $v_n = 1, v_{n+1} = -1,$ otherwise $v_j = 0$.