Journal of Biomechanics 51 (2017) 83–88

Contents lists available at ScienceDirect

Journal of Biomechanics

journal homepage: www.elsevier.com/locate/jbiomech
www.JBiomech.com

Improved reduced-order modelling of cerebrovascular flow distribution by accounting for arterial bifurcation pressure drops

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ARTICLE INFO

Article history:
Accepted 3 December 2016

Keywords:
0D model
Circle of Willis
Cerebral blood flow
Boundary conditions
Aneurysms

ABSTRACT

Reduced-order modelling offers the possibility to study global flow features in cardiovascular networks. In order to validate these models, previous studies have been conducted in which they compared 3D computational fluid dynamics simulations with reduced-order simulations. Discrepancies have been reported between the two methods. The loss of energy at the bifurcations is usually neglected and has been pointed out as a possible explanation for these discrepancies. We present distributed lumped models of cerebrovasculatures created automatically from 70 cerebrovascular networks segmented from 3D angiograms. The outflow rate repartitions predicted with and without modelling the energy loss at the bifurcations are compared against 3D simulations. When neglecting the energy loss at the bifurcations, the flow rates though the anterior cerebral arteries are overestimated by $4.7 \pm 6.8\%$ (error relative to the inlet flow rate, mean $\pm$ standard deviation), impacting the remaining volume of flow going to the other vessels. When the energy loss is modelled, this error is dropping to $0.1 \pm 3.2\%$. Overall, over the total of 337 outlet vessels, when the energy losses at the bifurcations are not modelled the 95% of agreement is in the range of $\pm 13.5\%$ and is down to $\pm 6.5\%$ when the energy losses are considered. With minimal input and computational resources, the presented method can estimate the outflow rates reliably. This study constitutes the largest validation of a reduced-order flow model against 3D simulations. The impact of the energy loss at the bifurcations is here demonstrated for cerebrovasculatures but can be applied to other physiological networks.

1. Introduction

In the past decade, 3D image-based simulations have been employed to characterize global and local blood flow features in the cardiovascular system (Taylor and Figueroa, 2009). Observations of the interactions between blood flow and vessels have opened new possibilities of biomechanical investigations. However, although 3D simulations allow comprehensive analysis, they come at a cost. Generating the required discretized models from medical images implies time-consuming pre-processing operations, and simulating the time-dependent 3D flows requires non-negligible time and computational resources.

Reduced-order modelling offers a good compromise between completeness and cost when only global flow features are of interest, and so constitute a perfect fit for modelling large cardiovascular networks (van de Vosse and Stergiopulos, 2011). These models offer multiple possibilities: they can serve as boundary conditions to effectively close the 3D Navier-Stokes equations (Formaggia et al., 2001; Liang et al., 2016; Marzo et al., 2011); they can be used to perform inverse modelling or sensitivity studies (Pant et al., 2014), to swiftly evaluate the consequences of different input conditions, pathological or surgical alterations, etc.; and, as highlighted in the present study, they can provide an important 'pre-flight' check to test the physiological plausibility of outflows for 3D CFD models when commonplace default pressure outlet boundary conditions are used.

Recurrent questions concern the accuracy of reduced-order model predictions, and how to validate them. While validation against in-vivo (Olufsen et al., 2000; Reymond et al., 2009b) or in-vitro (Bessems et al., 2008; Matthys et al., 2007) measurements is a natural choice, discrepancies between measurements and predictions due to the assumptions and simplifications of the reduced-order models can be difficult to differentiate from the inherent uncertainties of the measurements. 3D simulations are here a weapon of choice to isolate the influence of the assumptions made in reduced-order modelling since they are designed to mimic the 3D equations. Previous studies have followed this path and compared reduced-order models and 3D

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http://dx.doi.org/10.1016/j.jbiomech.2016.12.004
0021-9290/

simulations in arterial models (Schrauwen et al., 2014; Xiao et al., 2014) and more complex geometries such as the cerebrovasculature (Grinberg et al., 2011; Moore et al., 2005; Reymond et al., 2012).

These studies showed a good agreement between the two modelling techniques in the larger vessels where the flow is expected to be unidirectional and when the geometries remain simple. However, discrepancies regarding the pressures and flow rates have been observed in more complex configurations as in the cerebrovasculature (Grinberg et al., 2011; Moore et al., 2005; Reymond et al., 2012). On these geometries, one assumption is that part of these discrepancies arose from the lack of the model taking into account the loss of energy at the vessel bifurcations. To tackle this question directly, in the present study, we present a distributed lumped parameter model that incorporates a recently-developed approach to handle bifurcation pressure drops (Mynard and Valen-Sendstad, 2015), and perform a comparative study of its predictions against 3D simulations. The simulations are conducted for 70 subjects, using image-based models of their cerebrovasculature. The purpose of this study is thus twofold: (i) to validate the prediction of outflow divisions by reduced order models, using a comprehensive 3D CFD database as the gold standard; and (ii) to demonstrate, for the first time, the impact of accounting for pressure losses at bifurcations in reduced order models.

2. Materials and methods

2.1. Reduced order flow model

We consider a network constituted by connected rigid straight vessels bounded by nodes being inlets, outlets, or bifurcations. In this network, we assume a steady-state, fully developed, pressure-driven flow with a no-slip boundary condition. Thus, the Navier-Stokes equations for a Newtonian fluid boil down to the Hagen-Poiseuille equation, where the flow rate Q_k of each vessel k, is proportional to the pressure gradient along the vessel and the inverse of its resistance:

$$Q_k = \frac{\Delta P_k}{R_k}$$

In this study, two types of resistance are considered. For each vessel k, the hydraulic resistance is taken into account as,

$$R_k^h = \frac{8\mu L_k}{\pi r_k^4}$$

which is function of the vessel length L_k, its mean radius r_k and the dynamic viscosity of the fluid μ. In addition, for vessels originating from a bifurcation, a second resistance R_k^b is added to model the pressure drop induced by this geometrical element. The resistance is added in series and is modelled following the approach of Mynard and Valen-Sendstad (2015) as,

$$R_k^b = \frac{\rho Q_{dat}^2}{2\pi^2 r_{dat}^4}\left[\frac{1}{Q_k} + \frac{Q_k r_{dat}^4}{Q_{dat}^2 r_k^4} - \frac{2}{Q_k}\frac{r_{dat}^2}{r_k^2}\cos\left(\alpha_{(dat,k)}\right)\right]$$

where ρ is the density of the fluid, Q_{dat} and r_{dat} are respectively the flow rate and the radius of the datum supplier vessel at the network bifurcation. The angle $\alpha_{(dat,k)}$ is the angle between the datum supplier vessel and the considered vessel k, its definition being detailed in the next subsection. Each segment of the vascular network is thus discretized as one element modelled as an electric resistance using the described hydraulic analogy. Each segment has then its own relation with its pressure drop ΔP_k, its flow rate Q_k and its resistance $R_k = R_k^h$ or $R_k = R_k^h + R_k^b$ in the case of a bifurcation. By applying an analogue of Kirchhoff's current law to satisfy segment-to-segment mass conservation, the equations for all segments are assembled and result in a system of algebraic equations. Pressures are prescribed at the outlets, whereas a fixed flow rate is prescribed at the inlets of the network. This system is solved using the iterative process implemented in the open source pyNS (Manini et al., 2014).

2.2. Reduced-order representation of the vascular network

The construction of the system of equations depends on the parameters computed from the considered 3D vascular model. The centerlines are automatically computed thanks to VMTK (Antiga et al., 2008) and divided into a number of segments, as described in the previous subsection. Each piece of the centerlines is defined as N_i points in space linked by vectors δ_i, each point having an corresponding maximal-inscribed sphere radius computed via the associated Voronoi diagram, i.e. corresponding to the local minimum lumen radius (Antiga and Steinman, 2004). The complexity of the model is further reduced by computing the total segment length L_k and its mean radius r_k for each segment k of the network. The length is computed as

$$L_k = \sum_i ||\delta_i||_{L_2},$$

While the mean radius r_k is extracted from the sum of the hydraulic resistances of the segment δ_i,

$$r_k = \left(\frac{L}{C_k}\right)^{\frac{1}{4}},$$

where

$$C_k = \sum_i \frac{\delta_i}{r_i^4}.$$

Where r_i is the radius at the point i of the centerline. Branching angle at arterial junctions are kept in order to characterize the bifurcation resistance. We associate a unit vector to the datum supplier segment $\mathbf{n}_{dat}$ and, for each segment j of the junction connected to the supplier, a unit vector $\mathbf{n}_{j,k}$. These unit vectors are representing the direction of the centerlines directly upstream or downstream of the bifurcation, as illustrated in Fig. 1 (left figure). This step is automated and based on objective criteria aimed at generating robust results, c.f. (Antiga and Steinman, 2004) for technical details. For each bifurcation resistance R_k^b, the angle α is defined as the dot product between the vectors $\mathbf{n}_{dat}$ of the upstream vessel and $\mathbf{n}_{j,k}$ of the j-th downstream vessel at the junction.

2.3. Database of 3D CFD models

In order to test the accuracy of our reduced order model against the 3D models it is designed to approximate, we used the geometries and results from a large CFD study on middle cerebral artery (MCA) aneurysms. We were provided with 70 segmented multi-branch models extending from the proximal internal carotid artery (ICA) to distal MCA branches and their boundaries, e.g. flow rates and pressures at the inlet and outlets.

The lumen geometries were segmented from the 3D angiograms and the obtained geometries were discretized with tetrahedral meshes using linear elements, resulting in an average of 4.6 million elements. The Navier–Stokes equations were solved using standard numerical techniques (Ansys CFX, Ansys, Canonsburg, U.S.). Pulsatile flow boundary condition was applied at the inlet and time-dependent pressure boundary conditions were imposed at the outlets. The outlet

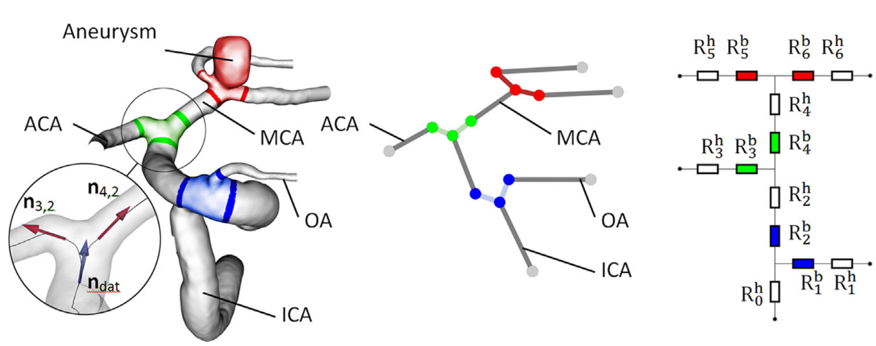

Fig. 1. **Left:** 3D cerebrovascular model, highlighting the bifurcations. The second bifurcation and its vectors are inset. Middle: Same model transformed into a network of edges and connecting nodes, note the bifurcation segments. Each edge k has the information needed to estimate the pressure drop in the network namely, length, equivalent radius and vectors at its extremities. Note that the model is flattened automatically by our software, allowing an easier visualization of the low order model next to its 3D counterpart. Right: the equivalent electrical circuit.