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An ultrasound elastography method to determine the local stiffness of arteries with guided circumferential waves

Guo-Yang Li[a,1], Qiong He[b,1], Guoqiang Xu[a], Lin Jia[a], Jianwen Luo[b], Yanping Cao[a,*]

[a] *Institute of Biomechanics and Medical Engineering, AML, Department of Engineering Mechanics, Tsinghua University, Beijing 100084, PR China*
[b] *Department of Biomedical Engineering, School of Medicine, Tsinghua University, Beijing 100084, PR China*

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ABSTRACT

Arterial stiffness is highly correlated with the functions of the artery and may serve as an important diagnostic criterion for some cardiovascular diseases. To date, it remains a challenge to quantitatively assess local arterial stiffness in a non-invasive manner. To address this challenge, we investigated the possibility of determining arterial stiffness using the guided circumferential wave (GCW) induced in the arterial wall by a focused acoustic radiation force. The theoretical model for the dispersion analysis of the GCW is presented, and a finite element model has been established to calculate the dispersion curve. Our results show that under described conditions, the dispersion relations of the GCW are basically independent of the curvature of the arterial wall and can be well-described using the Lamb wave (LW) model. Based on this conclusion, an inverse method is proposed to characterize the elastic modulus of artery. Both numerical experiments and phantom experiments had been performed to validate the proposed method. We show that our method can be applied to the cases in which the artery has local stenosis and/or the geometry of the artery cross-section is irregular; therefore, this method holds great potential for clinical use.

1. Introduction

Arterial stiffness has a close relationship with some cardiovascular diseases (Cecelja and Chowienczyk, 2012; Shirwany and Zou, 2010), and therefore, non-invasive measurement of arterial stiffness has received considerable attention in recent years. To date, methods based on an evaluation of the pulse wave velocity (Brands et al., 1998; Chubachi et al., 1994; Luo et al., 2012; Meinders et al., 2001; Huang et al., 2016), and the blood pressure induced static-strain within the arterial walls have been proposed by many authors to assess arterial stiffness (Hunter et al., 2010; Khamdaeng et al., 2012; Ribbers et al., 2007). However, in principle, it is difficult to quantitatively determine the local elastic properties of the arterial wall using these methods.

Recently, the shear wave elastography method has been investigated for measuring arterial stiffness (Bernal et al., 2011; Couade et al., 2010; Li et al., in press). This method relies on the use of the acoustic radiation force (ARF) (Sarvazyan et al., 1998) to generate broad-band guided axial wave (GAW) in the arterial wall. The dispersion relation of the GAW is then determined and used to deduce the arterial stiffness. This method is promising for instantaneously measuring arterial stiffness because the measurement can be conducted a number of times in each cardiac cycle, e.g., 13 times/cycle (Couade et al., 2010). However, it has been demonstrated that the dispersion relation of the GAW is significantly affected by the curvature of the arterial wall in the low-frequency range (typically 0–1000 Hz) (Li et al., in press; Li and Rose, 2006; Maksuti et al., 2016). In the literature, critical frequency has been suggested beyond which effects of curvature are negligible and the dispersion curve of the guided axial wave can be fitted with the dispersion curve given by the Lamb wave model (Maksuti et al., 2016). In a very recent study, Li et al. (in press) showed that such a critical frequency depends on both elastic properties and geometrical parameters of the arterial wall. Besides, it is difficult to apply the method based on the GAW to the cases in which the wall thickness and/or the inner radius of the artery vary along the axial direction and/or the cross-sectional geometry of the artery is irregular (e.g., non-circular). Considering that these cases are frequently encountered in clinics, the development of a robust method to measure the local arterial stiffness is still urgently needed.

Based on the above premise, this study aimed to develop a novel inverse approach based on the guided circumferential wave (GCW) to characterize the elastic properties of the arterial wall. To this end, a finite element (FE) model has been built to study the

* Corresponding author. Fax: +86 10 62781284.
E-mail address: caoyanping@tsinghua.edu.cn (Y. Cao).
[1] These authors made equal contribution to this study.

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dispersion relation of the GCW. Our FE results show that the GCW can be well-described with the Lamb wave (LW) model in the frequency range of interest in this study when the ratio of the inner radius of the arterial wall to its thickness is greater than a certain critical value. This finding enables us to develop a simple inverse method to measure the elastic properties of the arterial wall from the dispersion curve of the GCW induced in the artery by the focused ARF. Both numerical experiments and phantom experiments were performed to validate the inverse method. Furthermore, we show that our method is applicable to the cases in which the arterial wall has a local stenosis and/or the geometry of the artery cross-section is irregular, and therefore holds great potential for clinical use.

2. Theoretical model and FE simulations

Following previous studies (Bernal et al., 2011; Li et al., in press; Maksuti et al., 2016), the arterial wall was assumed to be a long hollow cylinder surrounded by fluid both inside and outside (see Fig. 1(a)) in the present study, and we generalized our analysis to the case in which the artery has a more general shape (Section 5). The guided wave that is of interest in this study is the so-called GCW (Liu and Qu, 1998a; Rose, 2014). Both the theoretical and the FE models that were used to analyze the GCW are presented below.

2.1. Basic equations for acoustic waves in solid and fluid media

The equilibrium equation for the elastic solid free of body force is

$$\mu\nabla^2\mathbf{u}+(\lambda+\mu)\nabla(\nabla\cdot\mathbf{u})=\rho\ddot{\mathbf{u}}, \tag{1}$$

where λ and μ are the *Lamé* constants, and ρ denotes the mass density of the elastic solid. $\ddot{\mathbf{u}}$ denotes $\partial^2\mathbf{u}/\partial t^2$, where t is time. The constitutive law for the linear elastic solid is $\boldsymbol{\sigma}=\mu(\nabla\mathbf{u}+\mathbf{u}\nabla)+\lambda(\nabla\cdot\mathbf{u})\mathbf{I}$. By decomposing the displacement $\mathbf{u}$ as $\mathbf{u}=\nabla\varphi+\nabla\times\boldsymbol{\psi}$, where $\nabla\cdot\boldsymbol{\psi}=0$ (Achenbach, 1973), and inserting this expression into Eq. (1), the following wave equation can be obtained

$$\begin{cases}\nabla^2\varphi=\frac{1}{c_l^2}\ddot{\varphi}\\ \nabla^2\boldsymbol{\psi}=\frac{1}{c_t^2}\ddot{\boldsymbol{\psi}}\end{cases}, \tag{2}$$

where $c_l=\sqrt{(\lambda+2\mu)/\rho}$ and $c_t=\sqrt{\mu/\rho}$ denote the velocities for the longitudinal and transverse waves in the elastic solid, respectively.

The fluid we considered here is a compressible and inviscid fluid, and the equilibrium equation for the small motion of the fluid can be given by

$$\nabla\left(\kappa\nabla\cdot\mathbf{u}^{\mathrm{F}}\right)=\rho^{\mathrm{F}}\ddot{\mathbf{u}}^{\mathrm{F}}, \tag{3}$$

where $\mathbf{u}^{\mathrm{F}}$ is the displacement of the fluid and κ and ρ^{F} denote the bulk modulus and mass density, respectively. Eq. (3) gives $\nabla\times\mathbf{u}^{\mathrm{F}}=0$; hence, we can define $\mathbf{u}^{\mathrm{F}}=\nabla\chi$ and then Eq. (3) can be rewritten as

$$\nabla^2\chi=\frac{1}{c_p^2}\ddot{\chi}, \tag{4}$$

where $c_p=\sqrt{\kappa/\rho^{\mathrm{F}}}$ is the velocity of the pressure wave in fluid. The pressure p induced by the deformation of the fluid is $p=-\kappa\nabla\cdot\mathbf{u}^{\mathrm{F}}=-\kappa\nabla^2\chi$.

2.2. Guided circumferential waves (GCWs)

We proceeded to consider the propagation of the GCW, which can be simplified as a plane strain problem. The solution domains are the elastic circular annulus and the surrounding fluid, as shown in Fig. 1(a). Here, we introduce the polar coordinates system in the analysis. The displacements of the elastic solid and the fluid have only two nonzero components, i.e., u_r and u_θ for the elastic solid and u_r^{F} and u_θ^{F} for the fluid. At the interfaces between the fluid and elastic solid, the following interfacial conditions (ICs) are specified

$$\begin{cases}\sigma_{rr}=-p\\ \sigma_{r\theta}=0\\ u_r=u_r^{\mathrm{F}}\end{cases}, \text{ at } r=R \text{ and } r=R+h, \tag{5}$$

where R and h denote the inner radius and wall thickness of the circular annulus, respectively. Considering the circumferential wave, which propagates along the θ-direction, we can write (Liu and Qu, 1998a; Rose, 2014)

$$\begin{cases}\varphi=\varphi_0(r)e^{i(k_\theta\theta-\omega t)}\\ \psi=\psi_0(r)e^{i(k_\theta\theta-\omega t)}\\ \chi^{\mathrm{In}}=\chi_0^{\mathrm{In}}(r)e^{i(k_\theta\theta-\omega t)}\\ \chi^{\mathrm{Out}}=\chi_0^{\mathrm{Out}}(r)e^{i(k_\theta\theta-\omega t)}\end{cases}, \tag{6}$$

where k_θ is the wave number and ω denotes the circular frequency, χ^{In} and χ^{Out} relate to the fluid region inside and outside of the circular annulus, respectively. The phase of the GCW is a constant along the radial direction (Liu and Qu, 1998a), and the phase velocity along a circle with radius r ($R\le r\le R+h$) is equal to

$$c(r)=r\frac{\omega}{k_\theta}. \tag{7}$$

In the present study, we are interested in the phase velocity at $r=R+h/2$, whereas previous studies have centered on the phase

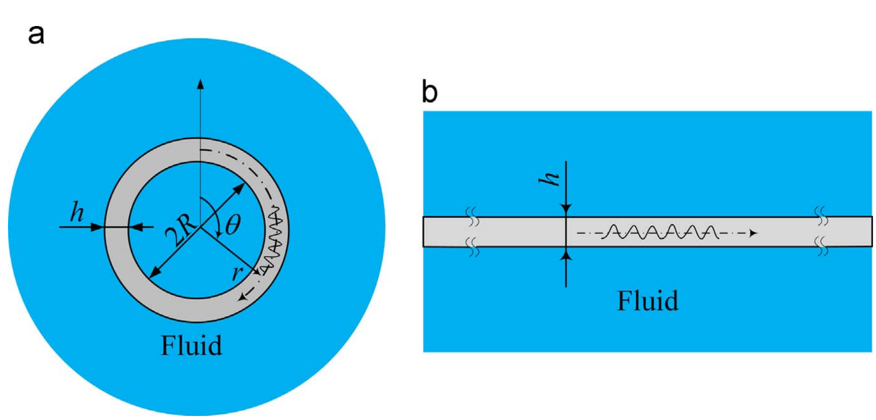

Fig. 1. The schematic diagram of (a) the GCW model and (b) the LW model.

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