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# Lateral migration of a capsule in a parabolic flow

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ABSTRACT

Red blood cells migrate to the center of the blood vessel in a process called axial migration, while other blood cells, such as white blood cells and platelets, are disproportionately found near the blood vessel wall. However, much is still unknown concerning the lateral migration of cells in the blood; the specific effect of hydrodynamic factors such as a wall or a shear gradient is still unclear. In this study, we investigate the lateral migration of a capsule using the boundary integral method, in order to compute exactly an infinite computational domain for an unbounded parabolic flow and a semi-infinite computational domain for a near-wall parabolic flow in the limit of Stokes flow. We show that the capsule lift velocity in an unbounded parabolic flow is linear with respect to the shear gradient, while the lift velocity in a near-wall parabolic flow is dependent on the distance to the wall. Then, using these relations, we give an estimation of the relative effect of the shear gradient as a function of channel width and distance between the capsule and the wall. This estimation can be used to determine cases in which the effect of the shear gradient or wall can be neglected; for example, the formation of the cell-free layer in blood vessels is determined to be unaffected by the magnitude of the shear gradient.



## 1. Introduction

The lateral migration of cells in blood flow has been a topic of extensive research since the time of Poiseuille (Sutera and Skalak, 1993). Red blood cells migrate to the center of the blood vessel in a process called axial migration, leading to the formation of a cell-free layer near the vessel wall; the resulting decrease in blood viscosity due to the presence of the cell-free layer is referred to as the Fahraeus–Lindqvist effect (Fahraeus and Lindqvist, 1931; Goldsmith et al., 1989). Conversely, experiments have found that other blood cells, such as white blood cells (Goldsmith and Spain, 1984; Phibbs, 1966; Schmid-Schonbein et al., 1980) and platelets (Leonard et al., 1972; Turitto et al., 1972; Turitto and Baumgartner, 1975) are disproportionately found near the blood vessel wall. Despite a large breadth of literature of research on the migration of cells within the blood, the quantifiable effect of factors such as red blood cell aggregation and hydrodynamic lift is still unclear.

In the microcirculation, where the vessel diameter is on the order of tens to hundreds of micrometers, the Reynolds number is less than $10^{-3}$, so blood flow in such vessels can be treated as a Stokes flow. In a Stokes flow, the lift of a deformable particle is induced by two hydrodynamic factors: the presence of a wall or shear gradient (Chan and Leal, 1979).

In terms of clarifying the effect of a wall, much research has been concentrated on the noninertial lateral migration of deformable particles in a simple shear flow near an infinite planar wall. Early analytical studies focused on the lateral migration of a liquid drop, or a small amount of liquid under the influence of surface tension suspended in a different liquid. These studies found that, at small deformation, the drop lift velocity is a function of the distance between the drop and the wall and the drop deformation (Chaffey et al., 1965; Chan and Leal, 1979; Smart and Leighton, 1991). Subsequent experimental and numerical studies confirmed these findings in cases when the drop is far from the wall, but found a decrease in the lift velocity relative to the predicted values when drops were placed near the wall (Smart and Leighton, 1991; Uijttewaal et al., 1993; Uijttewaal and Nijhof, 1995). Similar results have been reported for the lateral migration of vesicles, or liquid drops enclosed by a lipid bilayer (Smart and Leighton, 1991; Uijttewaal et al., 1993; Uijttewaal and Nijhof, 1995), and capsules, or liquid drops enclosed by an elastic membrane (Nix et al., 2014).

Other studies have examined the effect of a shear gradient on the lateral migration of deformable particles in large bounded or unbounded parabolic flows. Chan and Leal (1979) suggested that migration away from the flow centerline occurs when the viscosity ratio between the drop and the surrounding fluid takes on intermediate values, and that migration toward the flow centerline occurs at low or high viscosity ratios. While this behavior has been

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observed for a drop at finite Reynolds number (Mortazavi and Tryggvason, 2000), drops in Stokes flow are observed to migrate toward the centerline for a wide range of viscosity ratios (Griggs et al., 2007). Vesicles (Coupier et al., 2008; Kaoui et al., 2008) and capsules (Doddi and Bagchi, 2008; Helmy and Barthes-Biesel, 1982; Shi et al., 2012) in Stokes flow are also observed to migrate toward the centerline in both bounded and unbounded parabolic flows.

However, previous studies have not been able to clarify the separate effects of the wall and shear gradient on the lateral migration of deformable particles. Analytical studies are able to consider these two effects separately, but they are restricted to the assumptions of small deformation and large distances to the wall that rarely hold in realistic systems. Experimental and numerical studies are able to clarify the behavior of deformable particles at large deformation and small distances to the wall, but the effects of the wall and shear gradient are unable to be decoupled by experimental and most conventional numerical methods. Furthermore, an infinite or semi-infinite computational domain is necessary to isolate the effect of a shear gradient or single wall on the lateral migration of a particle.

Thus, in this study, we investigate the lateral migration of a capsule using the boundary integral method, in which infinite and semi-infinite computational domains are easily implemented, and the capsule velocity is represented as a sum of the surrounding flow and the boundaries within the flow, so the effects of the wall and shear gradient are easily decoupled. We show that the extent of the contribution of the shear gradient in a near-wall parabolic flow is determined solely by the distance between the wall and flow centerline and the distance between the capsule and wall. We also show the influence of viscosity ratio on the wall-induced and shear gradient-induced lift velocity and discuss the effect on the migration of red blood cells.

## 2. Numerical method

An initially spherical capsule of radius $a$ and inner viscosity $\lambda\mu$ is suspended in a Newtonian fluid with viscosity $\mu$. The capsule is suspended in a parabolic flow, with velocity oriented in the $x_1$ direction, that is either unbounded or bounded on one side by an infinite planar wall at $x_3=0$, as shown in Fig. 1. In either case, the parabolic flow takes on the form

$$\underline{u}^{\infty}=\left(\dot{\gamma}_w x_3-\frac{1}{2}kx_3^2\right)\underline{e}_1, \quad (1)$$

where $\underline{u}^{\infty}$ henceforth refers to the applied velocity field, $\dot{\gamma}_w$ is the shear rate at $x_3=0$,

$$k\equiv\frac{\mathrm{d}^2u_1^{\infty}}{\mathrm{d}x_3^2} \quad (2)$$

is the shear gradient, and $\underline{e}_i$ is the Cartesian basis. The capsule is placed at an initial position $x_3=h$ between $x_3=0$ and the

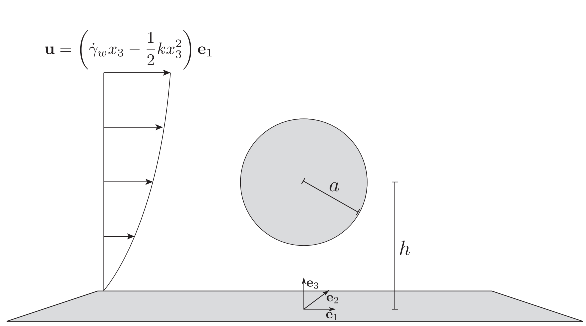


**Fig. 1.** Schematic showing capsule initial position and flow conditions in the case of a capsule placed near a planar wall.

centerline of the flow at $x_3=\dot{\gamma}_w/k$. The nondimensional shear rate at the capsule centroid is given by the local capillary number,

$$Ca_l=\frac{\mu\dot{\gamma}_l a}{G_s}, \quad (3)$$

where the local shear rate $\dot{\gamma}_l$ is defined in terms of $\dot{\gamma}_w$ as

$$\dot{\gamma}_l\equiv\frac{\mathrm{d}u_1^{\infty}}{\mathrm{d}x_3}=\dot{\gamma}_w-kh. \quad (4)$$

The capsule membrane has a surface shear modulus $G_s$ with negligible bending modulus, as the effect of the membrane elasticity is several orders of magnitude larger than the bending resistance (Parker and Winlove, 1999). The elasticity of the capsule membrane is described by the Skalak constitutive law (Skalak et al., 1973),

$$w_s=\frac{G_s}{4}[(\lambda_1^2-1)^2+(\lambda_2^2-1)^2+C(\lambda_1^2\lambda_2^2-1)^2], \quad (5)$$

where $\lambda_1$, and $\lambda_2$ are the principal stretch ratios of the membrane, and $C$ is a constant representing the degree of incompressibility of the capsule membrane, such that the area dilation modulus of the membrane is equal to $G_s(1+2C)$. Here, the values $C=1$ and C = 10 are considered. The shear gradient is nondimensionalised by the local shear rate to give the nondimensional parameter $ka/\dot{\gamma}_l$.

The capsule motion is solved numerically on Tesla C2070 and K20 graphics processing units (GPUs) (nVIDIA) using a method that couples the finite element method and boundary integral method (Matsunaga et al., 2014; Walter et al., 2010). In this method, the membrane tension is first solved using the constitutive law given in Eq. (5), then the viscous load on the membrane is solved using the finite element method to solve the equilibrium equation:

$$\int_A \underline{\hat{u}}\cdot\underline{q}\;\mathrm{d}A=\int_A \underline{\underline{\hat{\varepsilon}}}:\underline{\underline{T}}\;\mathrm{d}A, \quad (6)$$

where $\underline{q}$ is the viscous load, $\underline{\underline{T}}$ is the membrane tension, $\underline{\hat{u}}$ is a virtual displacement, and $\underline{\underline{\hat{\varepsilon}}}$ is the associated virtual strain.

The membrane velocity is solved from the viscous load using the boundary integral equation, given by

$$\begin{aligned} u_i(\underline{x}_0) \;&=\; u_i^{\infty}(\underline{x}_0)-\tfrac{1}{8\pi\mu}\textstyle\int_A G_{ij}(\underline{x}_0,\underline{x})q_j(\underline{x})\;\mathrm{d}A \\ &\quad+\tfrac{1-\lambda}{8\pi}\textstyle\int_A T_{ijk}(\underline{x}_0,\underline{x})u_j(\underline{x})n_k(\underline{x})\;\mathrm{d}A \end{aligned} \quad (7)$$

where $u_i(\underline{x}_0)$ and $u_i(\underline{x})$ are velocities on the capsule membrane, $\underline{n}$ is the outward-pointing normal vector of the area d$A$, and $G_{ij}$ is the Green's function with associated stress tensor $T_{ijk}$. In an unbounded flow, $G_{ij}$ is equal to the free-space Green's function,

$$G_{ij}^0(\underline{x}_0,\underline{x})=\frac{\delta_{ij}}{r}+\frac{r_ir_j}{r^3}, \quad (8)$$

where $r_i=x_{0,i}-x_i$ and $r=|\underline{r}|$. In cases where a planar wall is considered as part of the computational domain, the effect of the wall is solved using a modified Green's function (Blake, 1971),

$$\begin{aligned} G_{ij}(\underline{x}_0,\underline{x}) \;&=\; G_{ij}^0(\underline{x}_0,\underline{x})-G_{ij}^0(\underline{x}_0,\underline{x}') \\ &\quad+2\,h^2G_{ij}^D(\underline{x}_0,\underline{x}')-2\,hG_{ij}^{SD}(\underline{x}_0,\underline{x}'), \end{aligned} \quad (9)$$

where

$$G_{ij}^D(\underline{x}_0,\underline{x}')=(1-2\delta_{j3})\left(\frac{\delta_{ij}}{R^3}-\frac{3R_iR_j}{R^5}\right),$$

$$G_{ij}^{SD}(\underline{x}_0,\underline{x}')=(1-2\delta_{j3})\left(\frac{\delta_{ij}R_3-\delta_{i3}R_j+\delta_{j3}R_i}{R^3}-\frac{3R_iR_jR_3}{R^5}\right),$$

$\delta_{ij}$ is the Kronecker delta, $x_i'=x_i-2\delta_{i3}h$ is the position of a point on the capsule membrane reflected across a wall placed at $x_3=0$, $R_i=x_{0,i}-x_i'$, and $R=|\underline{R}|$.

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