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A physics based approach to the pulse wave velocity prediction in compliant arterial segments

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ABSTRACT

Pulse wave velocity (PWV) quantification commonly serves as a highly robust prognostic parameter being used in a preventative cardiovascular therapy. Being dependent on arterial elastance, it can serve as a marker of cardiovascular risk. Since it is influenced by a blood pressure (BP), the pertaining theory can lay the foundation in developing a technique for noninvasive blood pressure measurement. Previous studies have reported application of PWV, measured noninvasively, for both the estimation of arterial compliance and blood pressure, based on simplified physical or statistical models. A new theoretical model for pulse wave propagation in a compliant arterial segment is presented within the framework of pseudo-elastic deformation of biological tissue undergoing finite deformation. An essential ingredient is the dependence of results on nonlinear aspects of the model: convective fluid phenomena, hyperelastic constitutive relation, large deformation and a longitudinal pre-stress load. An exact analytical solution for PWV is presented as a function of pressure, flow and pseudo-elastic orthotropic parameters. Results from our model are compared with published *in-vivo* PWV measurements under diverse physiological conditions. Contributions of each of the nonlinearities are analyzed. It was found that the totally nonlinear model achieves the best match with the experimental data. To retrieve individual vascular information of a patient, the inverse problem of hemodynamics is presented, calculating local orthotropic hyperelastic properties of the arterial wall. The proposed technique can be used for non-invasive assessment of arterial elastance, and blood pressure using direct measurement of PWV, with account of hyperelastic orthotropic properties.

1. Introduction

Historically PWV has conceptually been based on the Moens–Korteweg equation, which predicts PWV based on an acoustical approach applied to a fluid structure interaction (FSI) of blood flow with a linear elastic cylindrical artery (O'Rourke, 2005). More accurate is a nonlinear traveling wave model developed for a compliant thin-walled linear elastic tube filled with an incompressible fluid (Liberson et al., 2014; Lillie et al., 2014). This model accounts for the convective ideal fluid flow interacting with a linearly elastic aortic vessel. However assumptions regarding linear deformation of an arterial vessel are not precisely applicable to living blood vessels as is proved in numerous related publications (Chuong and Fung, 1983; Cox, 1975; Demiray et al., 1988; Fung et al., 1979; Holzaphel, 2000; Holzaphel et al., 2000; Humphrey, 1995; Humphrey, 1999; O'Rourke, 2005; Takamizawa and Hayashi, 1987; Vaishnav et al., 1972; Zhou and Fung, 1997). Nonlinearities in blood vessel elasticity and finite deformation are vitally important to match static testing results for an internal pressure load and an axial tension of a cylindrical artery under physiological conditions (Cox, 1975; Demiray et al., 1988; Fung et al., 1979; Zhou and Fung, 1997).

The link between PWV, or pulse transit time (PTT), and blood pressure (BP) has been previously investigated based on statistical regression models, or empirical representation of an incremental isotropic elastic modulus as a function of a transmural pressure (Chen et al., 2000, 2009; Muehlsteff and Schuett, 2006; Zhang et al., 2009). We propose a physics based mathematical model that predicts the general relationship between PWV and BP with a rigorous account of nonlinearities in the fluid dynamics model, blood vessel elasticity, finite deformation of a thin anisotropic wall and a longitudinal pre-stress. The combined effect of BP and blood flow on PWV is derived and correlates with clinical evidence presented in Nurnberger et al. (2003) and Salvi et al. (2013). In order to retrieve individual vascular information of a patient, we present the PWV based solution for the inverse problem of hemodynamics, calibrating individual orthotropic hyperelastic arterial properties based on typical diagnostic measurements and

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Nomenclature

PWV	Pulse wave velocity (m/s)
FSI	Fluid structure interaction
BP	Blood pressure
A	Cross sectional area (m^2)
u	Axial flow velocity (m/s)
p	Transmural pressure (Pa)
ρ	Density of incompressible fluid (kg/m^3)
R,r	Internal wall radii in a zero stress and loaded conditions respectively (m)
η	Ratio of the wall deflection to R
C_{MK}	Moens–Korteweg speed of propagation (m/s)
H, h	Thickness of the wall at a zero stress and loaded conditions respectively (m)
F	Axial pretension force (N)
σ_θ, σ_z	Circumferential and axial Cauchy stress components (Pa)
E_θ, E_z	Circumferential and axial Green-Lagrange strain components
A	$\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}$ symmetric tensor of material constants (Pa)

Subscripts

(r,θ,z)	Radial, circumferential and axial components of a corresponding 3D vector

an optimization technique. This study is a continuation of the previous work (Liberson et al., 2014; Lillie et al., 2014) in the context of *in-vivo* validation and application of the proposed methodology to continuous, noninvasive blood pressure measurements. The proposed technique can be used for non-invasive assessment of arterial elastance, directly related to the direct measurement of PWV, with account of hyperelastic orthotropic properties of a biological vessel.

2. Theory

2.1. Dynamics of incompressible flow in a compliant vessel

One dimensional models simulating blood flow in arteries effectively describe pulsatile flow in terms of averages across the section flow parameters. Although they are not able to provide the details of flow separation, recirculation, or shear stress analysis, they should accurately represent the overall and average pulsatile flow characteristics, particularly PWV. Derivations of one dimensional models can be found in a number of papers, see for instance Cascaval (2012), Liberson et al. (2014), and Lillie et al. (2014), and are not repeated here.

Conservation of mass and momentum results in the following system of one dimensional equations

$$\frac{\partial A}{\partial t}+\frac{\partial}{\partial z}(uA)=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\frac{\partial}{\partial z}\left(\frac{u^2}{2}+\frac{p}{\rho}\right)=0 \tag{2}$$

The model described by (1) and (2) is applied exclusively to the large arteries, where viscosity is important only in the regions of a boundary layer, and does not practically have an impact on PWV (Caro et al., 2012; Parkhusrt et al., 2012). As it follows from monographs of Caro et al. (2012) and Li et al. (1981) the introduction of visco-elastic, rather than elastic vessel wall properties results in a slight increase in wave speed, but affects noticeably the attenuation of a waveform.

For an impermeable thin walled membrane, neglecting inertia forces, the vessel pressure–strain relationship is maintained by equilibrium condition as a function $p=p(\eta)$, based on relevant constitutive relations. Noting that $A=\pi R^2(1+\eta)^2$, and assuming that transmural pressure is a smooth function of a wall normal deflection (derivative $p_\eta=\partial p/\partial\eta$ exists at any point), the total system of equations can be presented in the following non-conservative form

$$\frac{\partial U}{\partial t}+H(U)\frac{\partial U}{\partial z}=0 \tag{3}$$

where

$$U=\begin{bmatrix}\eta\\ u\end{bmatrix};\ H=\begin{bmatrix} u & \frac{1+\eta}{2}\\ \frac{p_\eta}{\rho} & u\end{bmatrix} \tag{4}$$

We find the eigenvalues of $H(U)$ to be real and distinct. PWV is associated with the forward running wave velocity, i.e. the largest eigenvalue (Hirsch, 2006), hence it is identified as

$$\mathrm{PWV}=u+\sqrt{\frac{1+\eta}{2\rho}p_\eta} \tag{5}$$

The partial derivative P_η indicates sensitivity of pressure with respect to the wall normal deflection η, and has a clear interpretation as tangent (incremental) moduli in finite strain inelasticity. System (1) and (2) is typically closed by defining an explicit algebraic relationship between pressure and normal deflection. For instance, in case of a small deformation and linear elastic response, where E is the Young's modulus, ν – the Poisson coefficient, and pressure relates to the circumferential strain η via

$$p=\frac{\overline{E}H}{R}\eta,\ \overline{E}=\frac{E}{1-\nu^2} \tag{6}$$

Eqs. (4) and (5) can be transformed to the simplified form derived differently in Cascaval (2012), Liberson et al. (2014), and Lillie et al. (2014),

$$\mathrm{PWV}=u+c_{MK}\sqrt{1+\eta},\ c_{MK}=\sqrt{\frac{\overline{E}h}{2\rho R}} \tag{7}$$

Under the assumption $u\ll c_{MK},\eta\ll 1$ (linearized approach) Eq. (7) converts into the Moens–Korteweg equation for the forward and backward traveling waves. In the general case, Eq. (5) should be supplemented by appropriate constituent equations for a hyperelastic anisotropic arterial wall, accounting for finite deformation.

2.2. Hyperelasticity of the vessel wall

It is assumed that arterial wall is hyperelastic, incompressible, anisotropic, and undergoing finite deformation. After a few original loading cycles (preconditioning) the arterial behavior follows some repeatable, hysteresis free pattern with a typical exponential stiffening effect regarded as pseudo elastic (Fung et al., 1979; Holzaphel et al., 2000). Numerous formulations of constitutive models for arteries have been proposed based on polynomials

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