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On the halt of spontaneous capillary flows in diverging open channels

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ABSTRACT

Due to their compactness and independence of exterior energy sources, capillary microsystems are increasingly used in many different scientific domains, from biotechnology to medicine and biology, chemistry, energy and space. Obtaining a capillary flow depends on channel geometry and contact angle. A general condition for the establishment of a spontaneous capillary flow in a uniform cross section channel has already been derived from Gibbs free energy.

In this work, we consider spontaneous capillary flows (SCF) in diverging open rectangular channels and suspended channels, and we show that they do not flow indefinitely but stop at some location in the channel.

In the case of linearly diverging open channels, we derive the expression that determines the location where the flow stops. The theoretical approach is verified by using the Surface Evolver numerical program and is checked by experiments.

The approach is extended to sudden enlargements, and it is shown that the enlargements can act as stop and trigger valves.

1. Introduction

Capillary microsystems are increasingly used in many different scientific domains, from biotechnology to biology and medicine, chemistry, thermics, energy and space. In such systems the energy source that is responsible for the flow is the surface energy of the walls. No external, bulky energy sources and no moving off-chip parts are needed, which make such systems compact and easily portable. This is the case of "point-of-care" or "personal self testing" systems in biotechnology and medicine, which enable patients to easily monitor in their home or at the doctor's office their physical condition and adjust their treatment without having to go to an analysis laboratory [1–4]. This is also the case of "vanes" used in space to guide fluids in zero-gravity environment [5]. In the energy domain, capillarity is now widely used in fuel cell technology [6,7], and in thermics, open capillary flows are used for the cooling of heated elements [8].

From a general standpoint, obtaining a capillary flow depends on channel geometry and contact angle. A general condition for the establishment of a spontaneous capillary flow (SCF) in a uniform cross section channel has been derived from Gibbs free energy [9]. This relation is valid for closed or open channels, however it is restricted to uniform cross section channels. We extend here this relation to the case of non-uniform cross section channels.

In this work, we first consider SCFs in linearly diverging open rectangular channels and suspended channels, and we show that they do not flow indefinitely but stop at some location in the channel. In a similar approach to that of uniform channels, using Gibbs free energy, we derive the expression that determines the location where the flow stops. The theoretical approach is verified by using the Surface Evolver numerical program [10]. Experiments using suspended diverging channels are reported here that agree with the theory.

In a second step, we extend the theory to large widening angles, and we consider suddenly diverging open rectangular channels and suspended channels. It is shown that such devices may

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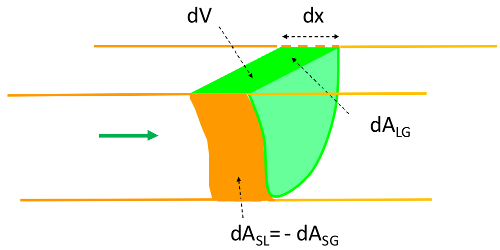

Fig. 1. Sketch of the liquid front advancing along the solid surface by capillarity.

or may not stop the capillary flow, depending on their geometry. It is demonstrated that such geometrical features can act as stop or trigger valves, in a same manner as used in conventional closed systems [11,12].

2. Theory

First, let us recall the condition for SCF in a uniform cross section channel for an isothermal flow [9,13]. The morphology of the free interface is such that it evolves to reduce the Gibbs free energy G

$$dG = \gamma \, dA - p \, dV < 0, \tag{1}$$

where γ denotes the surface tension, A the surface area, p the pressure and V the volume.

By definition, SCF occurs in a capillaric channel where the upstream pressure is null, and SCF occurs as long as the pressure at the flow front ($p<0$) is smaller than that of the reservoir ($p \sim 0$). In the case of an open rectangular channel or suspended channel, using Young's law, it can be shown that Eq. (1) can be cast in the form

$$\frac{dA_{LG}}{dA_{SL}} < \cos\theta, \tag{2}$$

where θ is the contact angle, A_{LG} the liquid–gas surface area, and A_{SL} the liquid–solid surface area, as shown in Fig. 1. In this figure, the real shape of the interfaces has been simplified; however, it was shown in [9] that this simplification does not affect the validity of the reasoning.

Let us now consider linearly diverging open rectangular channels, or linearly diverging suspended channels, as sketched in Fig. 2.

Assume an infinitesimal progression dx of the front of the fluid. For the open rectangular channel, the change in liquid–solid surface area is

$$dA_{SL} = 2h\frac{dx}{\cos\alpha} + dx\,(w + dw), \tag{3}$$

and the change in liquid–air surface area is

$$dA_{LG} = \; dx\;(w + dw) + 2h\,dw, \tag{4}$$

where

$$dw = dx\,\tan\alpha. \tag{5}$$

Hence

$$\begin{aligned} dA_{SL} &= 2h\frac{dx}{\cos\alpha} + dx\,(w + dx\,\tan\alpha) \\ dA_{LG} &= dx\;(w + dx\,\tan\alpha) + 2h\,dx\,\tan\alpha \end{aligned} \tag{6}$$

Substitution of (6) in (2), neglecting second order terms, and noting $q = w/h$, yields

$$\frac{dA_{LG}}{dA_{SL}} = \frac{dx\;(w + dx\,\tan\alpha) + 2h\,dx\,\tan\alpha}{2h\,\frac{dx}{\cos\alpha} + dx\,(w + dx\,\tan\alpha)}$$

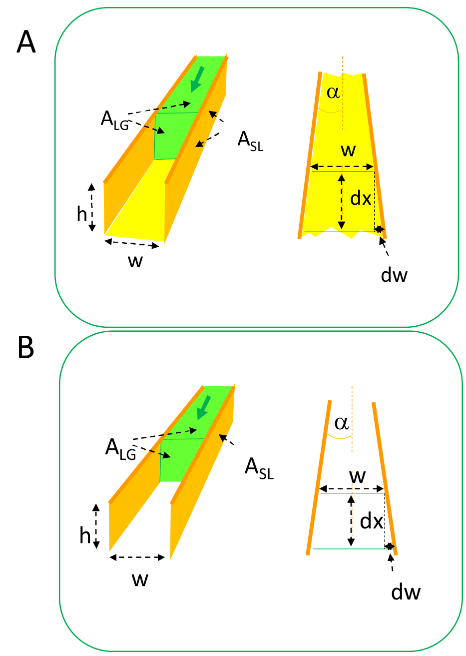

Fig. 2. Sketch of diverging channels of diverging angle 2α, the liquid front advancing along the solid surface by capillarity: A, open rectangular channel, B, suspended channel. It is shown in [9] that the real shape of the advancing interface does not modify the approach.

$$\begin{aligned} &\approx \frac{w + 2h\,\tan\alpha}{w + \frac{2h}{\cos\alpha}} \\ &= \frac{w\cos\alpha + 2h\,\sin\alpha}{w\cos\alpha + 2h} \quad < \cos\theta \end{aligned} \tag{7}$$

Finally, one obtains the SCF condition for a diverging suspended spontaneous capillary flow

$$\frac{w}{h} = q < \frac{2\,(\cos\theta - \sin\alpha)}{\cos\alpha\;(1 - \cos\theta)}. \tag{8}$$

Remark that if $\alpha = 0$, relation (8) collapses to the parallel walls condition [9]

$$\frac{w}{h} = q < \frac{2\;\cos\theta}{(1 - \cos\theta)}. \tag{9}$$

On the other hand, when α is small, relation (8) can be simplified by using first order terms of Taylor expansions for cos α and sin α

$$\begin{aligned} \frac{w}{h} &< \frac{2\,(\cos\theta - \alpha)}{(1 - \cos\theta)} \\ &= \frac{2\;\cos\theta}{(1 - \cos\theta)} - \frac{2\,\alpha}{(1 - \cos\theta)} \end{aligned} \tag{10}$$

In this form, it is shown that the condition for SCF is slightly stricter for diverging open rectangular channel than the expression based on the uniform cross section open rectangular channel

$$\left.\frac{w}{h}\right|_{diverging} < \left.\frac{w}{h}\right|_{uniform} - \frac{2\,\alpha}{(1 - \cos\theta)}. \tag{11}$$

In consequence, the location where SCF stops is located before that predicted by (9). Note that the case where $\theta < 45°$ must be set apart because of the formation of capillary filaments [14,15].

A similar approach can be done for suspended channels. Using (2) again, it is straightforward to show that the SCF condition is

$$\frac{w}{h} < \frac{\cos\theta - \sin\alpha}{\cos\alpha}. \tag{12}$$