ARTICLE IN PRESS
JID: JJBE [m5G;October 8, 2016;14:7]

Medical Engineering and Physics 000 (2016) 1–10

Contents lists available at ScienceDirect

Medical Engineering and Physics

journal homepage: www.elsevier.com/locate/medengphy

A theoretical computerized study for the electrical conductivity of arterial pulsatile blood flow by an elastic tube model

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ARTICLE INFO

Article history:
Received 4 February 2016
Revised 25 August 2016
Accepted 23 September 2016
Available online xxx

Keywords:
Theoretical modeling
Electrical conductivity
Arterial pulsatile blood flow
Elastic tube
Rigid tube

ABSTRACT

The electrical conductivity of pulsatile blood flow in arteries is an important factor for the application of the electrical impedance measurement system in clinical settings. The electrical conductivity of pulsatile blood flow depends not only on blood-flow-induced *red blood cell* (RBC) orientation and deformation but also on artery wall motion. Numerous studies have investigated the conductivity of pulsatile blood based on a rigid tube model, in which the effects of wall motion on blood conductivity are not considered. In this study, integrating Ling and Atabek's local flow theory and Maxwell–Fricke theory, we develop an elastic tube model to explore the effects of wall motion as well as blood flow velocity on blood conductivity. The simulation results suggest that wall motion, rather than blood flow velocity, is the primary factor that affects the conductivity of flowing blood in arteries.

1. Introduction

Over the past few decades, a number of investigations have demonstrated that the electrical conductivity of pulsatile blood flow in arteries is an important player for electrical impedance measurement system applications in clinical settings [1,19,21,24,27]. The changes in blood electrical conductivity are reported to mainly result from blood-flow-induced orientation and deformation of *red blood cells* (RBCs) [6,8,18,22]. In addition, it has been demonstrated that the characteristics of blood flow, e.g., steady flow or pulsatile flow in the artery, are critical in determining the electrical conductivity of arterial blood flow.

The effects of steady flow on the variation in electrical conductivity of blood have been well documented since the 1970s [6,8,18,22]. These researchers found that the change in electrical conductivity is primarily influenced by the orientation and deformation of RBCs; both are determined by fluid shear stress during steady flow [13,23]. Then, Gaw et al. [9–11] first reported the impact of pulsatile blood flow on electrical conductivity in a straight rigid model using the Womersley theory [28]. This study also found that the variation in electrical conductivity is affected by RBC orientation and deformation. The deformation of RBCs is determined by shear stress, while the orientation ratio of RBCs is determined by the shear rate during pulsatile flow.

It should be noted that all of these previous investigations were based on the assumption that the arterial wall is rigid [9–11,13,17,23]; however, this is not true under physiological conditions. In in vivo arteries, arterial walls undergo elastic deformations. The interaction between wall motion and pulsatile blood flow may lead to significant nonlinear effects on pulsatile blood flow and shear stress [2,4,14,15,25,26,29]; this interaction definitely influences RBC orientation and deformation, thus changing arterial electrical conductivity. However, to date, the quantitative relationship between electrical conductivity, wall motion, and pulsatile blood flow dynamics still remains elusive.

In this paper, we attempt to explore the relationship between electrical conductivity, wall motion, and pulsatile blood flow dynamics by proposing an elastic tube model and analyzing the effect of wall motion on pulsatile blood flow and arterial pulsatile blood flow conductivity. The elastic tube model integrates Ling and Atabek's 'local flow theory' [15] and the Maxwell–Fricke theory [7] and focuses on the orientation and deformation of ellipsoidal particles induced by shear stresses [9–11,13]. Moreover, particular attention is paid to clarifying the contribution of the arterial radius as well as the axial center-line velocity to blood conductivity; these can easily be measured by the Doppler ultrasonic technique [16].

2. Proposed elastic tube model

This section first presents a proposed elastic tube model for the electrical conductivity of arterial pulsatile blood flow. Then, we model the conductivity changes as functions of wall motion,

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http://dx.doi.org/10.1016/j.medengphy.2016.09.013
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blood-flow-induced RBC orientation and deformation and finally give the equations and numerical simulation methods.

2.1. Elastic tube model and equations

2.1.1. Pulsatile blood flow dynamics

Pulsatile blood flow in a straight and circular artery can be modeled as a homogeneous, incompressible Newtonian fluid flow in an isotropic, thin-walled, elastic tube with a longitudinal constraint. The continuity and Navier–Stokes equations governing pulsatile blood flow can be simplified as follows [2,15,25,28]

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial x} + \frac{\eta}{\rho}\cdot\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial u}{\partial r}\right) \tag{1}$$

$$\frac{\partial u}{\partial x} + \frac{1}{r}\cdot\frac{\partial (rv)}{\partial r} = 0 \tag{2}$$

with boundary conditions of

$$\left.\frac{\partial u}{\partial r}\right|_{r=0} = 0,\ v|_{r=0} = 0 \tag{3}$$

$$u|_{r=R} = 0,\ v|_{r=R} = \frac{\partial R}{\partial t} \tag{4}$$

where u and v denote the axial velocity and radial velocity, respectively; p is blood pressure; ρ and η denote the density and viscosity of blood, respectively; t is a symbol of time; x and r are the longitudinal and radial coordinates, respectively; and R is the arterial inner radius.

By introducing a relative radial coordinate $y = r/R$ into Eq. (1), the axial velocity in the straight artery has the following radial distribution

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\cdot\frac{\partial p}{\partial x} + \frac{\eta}{\rho}\cdot\frac{1}{R^2}\cdot\left(\frac{\partial^2 u}{\partial y^2} + \frac{1}{y}\cdot\frac{\partial u}{\partial y}\right) + \frac{1}{R}\cdot\frac{\partial u}{\partial y}\cdot\left(y\cdot\frac{\partial R}{\partial t} - v\right) + \frac{u}{R\cdot y}\cdot\left(v + y\cdot\frac{\partial v}{\partial y}\right) \tag{5}$$

Let us introduce Ling and Atabek's 'local flow theory' [15], that is, assuming that a small variation in x does not change the shape of the axial velocity profiles, the longitudinal gradient of the axial velocity u satisfies:

$$\frac{\partial u}{\partial x} = f(x,t)\cdot|u(x,y,t)| \tag{6}$$

Thus, by introducing Eq. (6) into Eq. (2), the radial velocity can be expressed as follows:

$$v = \frac{\partial R}{\partial x}\cdot\left(yu - \frac{2}{y}\cdot\int_0^y yudy\right) - \frac{R}{y}\cdot f(x,t)\cdot\int_0^y y|u|dy \tag{7}$$

In Eq. (7), $\frac{\partial R}{\partial x}$ can be expressed by

$$\frac{\partial R}{\partial x} = \left(\frac{\partial R}{\partial p}\right)\cdot\left(\frac{\partial p}{\partial x}\right) \tag{8}$$

where $\frac{\partial R}{\partial p}$ is determined by the arterial elastic properties and geometry of the arterial partial zero-stress state that can be measured by the pressure waveform $p(t)$ and radius waveform $R(t)$ using a Doppler ultrasonic instrument.

The pressure gradient can be derived from Eqs. (5)–(8), as follows [5]:

$$\frac{\partial p}{\partial x} = \frac{-\frac{\partial u_c}{\partial t} + \frac{u_c\cdot|u_c|}{R\cdot\int_0^1 y|u_c|dy}\cdot\left(\frac{\partial R}{\partial t}\right) + \frac{2}{R^2}\cdot\frac{\eta}{\rho}\cdot\left(\frac{\partial^2 u_c}{\partial y^2}\right)_{y=0}}{\frac{1}{\rho} - 2\cdot\frac{\partial R}{\partial p}\cdot\frac{u_c}{R}\cdot\frac{\int_0^1 yudy}{y|u_c|}\cdot u_c} \tag{9}$$

where u_c is the center-line blood velocity at $y = 0$.

The radial velocity v can be deduced from Eqs. (7)–(9), as follows:

$$v = \frac{\partial R}{\partial p}\cdot\frac{\partial p}{\partial x}\left[yu - \frac{2}{y}\cdot\left(\int_0^y yudy\right) - \frac{\int_0^1 yudy}{\int_0^1 y|u|dy}\cdot\int_0^y y|u|dy\right] + \frac{1}{y}\cdot\frac{\partial R}{\partial t}\cdot\frac{\int_0^y y|u|dy}{\int_0^1 y|u|dy} \tag{10}$$

The axial velocity u and radial velocity v can be deduced from Eqs. (5), (9) and (10). Then, the axial shear stress τ_{rx} and radial shear stress τ_{rr} are expressed as:

$$\tau_{rx} = \frac{\eta}{R}\cdot\frac{\partial u}{\partial y} \tag{11}$$

$$\tau_{rr} = 2\cdot\frac{\eta}{R}\cdot\frac{\partial v}{\partial y} \tag{12}$$

Because the radial shear stress τ_{rr} is far smaller than the axial shear stress τ_{rx}, i.e., $\tau_{rr} < <\tau_{rx}$, the total shear stress τ at time t is approximated as:

$$\tau \approx \tau_{rx} = \frac{\eta}{R}\cdot\frac{\partial u}{\partial y} \tag{13}$$

For the special case when wall motion is ignored, the elastic tube model is degenerated as a rigid tube model. The shear stress τ_{rg} in the rigid tube model is then expressed as follows (after [16]):

$$\tau_{rg}(y,t) = \frac{\eta}{\bar{R}}\sum_{n=-\infty}^{+\infty}\frac{\alpha_n j^{\frac{3}{2}} J_1(\alpha_n j^{\frac{3}{2}} y)}{J_0(\alpha_n j^{\frac{3}{2}}) - 1}u(0,\omega_n)e^{j\omega_n t} \tag{14}$$

where n is the harmonic number; J_0 and J_1 are the 0th-order and 1th-order Bessel functions of the first kind, respectively; $j = \sqrt{-1}$; $\bar{R}$ is the time-averaged arterial inner radius over one cardiac cycle; $\alpha_n = \bar{R}\sqrt{\rho\omega_n/\eta}$ is the Womersley number; $\omega_n = 2n\pi f$ is the angular frequency; f is the base frequency; and $u(0, \omega_n)$ is the nth harmonic component of the measured center-line velocity $u_c(t)$ and satisfies

$$u_c(t) = \sum_{n=-\infty}^{+\infty} u(0,\omega_n)e^{j\omega_n t} \tag{15}$$

Eqs. (14) and (15) describe the relationship between the shear stress τ_{rg} and center-line velocity $u_c(t)$ in the rigid tube model. Once the center-line velocity $u_c(t)$ and time-averaged radius $\bar{R}$ are given, the frequency components $u(0, \omega_n)$ of $u_c(t)$ are calculated by Eq. (15), and the shear stress τ_{rg} is then calculated using Eq. (14).

2.1.2. Conductivity of blood

In this study, flowing blood with insulating RBCs is modeled as a flowing insulating suspension with ellipsoidal particles. The bulk conductivity $\sigma_{bl}(t)$ of flowing blood is calculated as follows (after [11,13]):

$$\sigma_{bl}(t) = \frac{2}{\bar{R}^2}\cdot\int_0^{\bar{R}}\sigma_c(r,t)rdr \tag{16}$$

where $\sigma_c(r, t)$ is the conductivity of blood at any radial coordinate r that can be calculated by the Maxwell–Fricke Eq. [7]:

$$\frac{\sigma_c}{\sigma_p} = \frac{1-H}{1+(C-1)\cdot H} \tag{17}$$

where σ_p is the conductivity of the plasma; H is the haematocrit expressed as the volume fraction of RBCs relative to the total blood volume; and C is a factor that depends on the geometry and orientation of the RBC and is affected by shear stress.

The expressions for factor C can be found in the literature [3,11,13]. For an easy reference, these expressions are also presented as follows:

$$C = f(r)\cdot C_b + (1 - f(r))\cdot C_r \tag{18}$$