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Catastrophes and *ex post* shadow prices—How the value of the last fish in a lake is infinity and why we should not care (much)☆

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A B S T R A C T

Catastrophic risk is currently a hotly debated topic. This paper contributes to this debate by showing two results. First, it is shown that for a certain class of optimal control problems, the derivative of the value function with respect to the initial state may approach infinity as the state variable goes to zero, even when the first-order partial derivatives of the integrand and transition functions are finite. In the process, it is shown that standard phase diagrams used in optimal control theory contain more information than generally recognized and that the value function itself may be easily illustrated in these diagrams. Second, we show that even if the value function has an infinite derivative at some point, it is not correct to avoid this point in finite time at almost any cost. The results are illustrated in a simple linear-quadratic fisheries model and proven for a more general class of growth functions.

1. Introduction

The Economic Management of Catastrophic Risk (EMCR) is currently hotly debated, and several issues have not been resolved. Catastrophic risk is usually discussed in a dynamic framework in which the catastrophe is some detrimental event whose probability of occurrence is distributed over time. EMCR often applies dynamic optimization techniques in order to derive management rules. The standard approach to solving such problems is to first calculate the *ex post* solution, which is a contingency plan for optimal management after a catastrophe has occurred. Using the *ex post* solution as a building block, one then solves the *ex ante* management problem, which is the optimal program to be followed as long as the catastrophe does not occur (Clarke and Reed, 1994; Tsur and Zemel, 1995; Tsur and Zemel, 1998).

The present paper aims to elaborate on how the marginal value of a resource *after* a catastrophe, termed *ex post* shadow price, affects optimal management *before* the catastrophe. Two important results are derived that are the main theoretical contributions of this paper. First, it is shown that *ex post* shadow prices can become infinite even if all of the first-order

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partial derivatives of the integrand and transition functions have finite values on their domains. Second, it is shown that *ex post* shadow prices have limited or no impact on *ex ante* regulation if a catastrophe is defined as a total collapse in a resource stock.

The importance of *ex post* shadow prices for optimal management is analyzed in a number of papers. For example, de Zeeuw and Zemel (2012) analyze a pollution control problem in a dynamic programming framework and find that *ex post* shadow prices are important for *ex ante* regulation. Polasky et al. (2011) find the same in a fisheries model. As shown below, the results in these papers are caused by the authors not specifying the catastrophe as severe enough. When a catastrophe is defined as a total collapse in a resource, the *ex post* shadow price will, in general, be of no or relatively minor importance for optimal management before the catastrophe.

There seems to be a certain amount of confusion about how *ex post* shadow prices are generated and affect management. A prominent example is Weitzman's (2009) contribution. This paper has been much debated since its publication. The "dismal theorem" presented therein states that under certain conditions, the process of gathering data about a stochastic process leads to a situation where expected marginal utility explodes. Weitzman is careful to state that his result does not depend on "a mathematically illegitimate use of the symbol +∞" but arises naturally and that criticism of his results by "somehow discrediting this application of expected utility on the narrow grounds that infinites are not allowed in a legitimate theory of choice under uncertainty" is unfounded. This paper will show that infinite marginal values are most certainly allowed in such a theory, as they, under certain conditions, turn up endogenously. Thus, the data gathering process applied by Weitzman (2009) is not required to generate infinite shadow prices, even when all functional forms have everywhere finite derivatives. The realism of Weitzman's results have been challenged by, for example, Nordhaus (2011), Pindyck (2011) and Costello et al. (2010).

The question is then if infinite shadow prices or infinite marginal utility actually matter. Weitzman himself argues that "The burden of proof if in climate change is presumably upon whoever calculates expected discounted utilities without considering that structural uncertainty might matter more than discounting or pure risk. Such middle-of-the-distribution modeler should be prepared to explain why the bad fat tail of the posterior predictive PDF does not play a significant role in climate-change CBA when it is combined with a specification that assigns high disutility to high temperatures." This statement must be qualified. The dismal theorem refers to expected *marginal* utilities and not the *level* of disutility. However, even so, the dismal theorem seems to indicate that getting into a situation where marginal utility is infinity should be avoided at almost any cost. Indeed, when Inada conditions specify that functions evaluated at zero have infinite derivatives, the purpose is to ensure that a model economy converges to an interior steady state. One important implication of the present paper is that Inada conditions are not required to guarantee such an outcome as infinite shadow prices turn up endogenously. This property to some extent carries over to stochastic models, as it has been known since the work of Brock and Mirman (1972) that optimal paths converge to a uniquely non-trivial stationary solution and that this result depends on imposing Inada-conditions. Kamihigashi (2006) shows that if Inada conditions are not imposed, there will almost surely be convergence to zero. One could therefore be excused for thinking that imposing Inada conditions guarantee against it ever being optimal to reach a state with infinite shadow price, typically a state where the amount of some valuable variable is equal to zero. Mirman and Zilcha (1976) provide an example where consumption is not bounded away from zero even if Inada-conditions are imposed. It is shown in the following that this turns out to be wrong in models where a catastrophe is defined as a total collapse in the state variable. The Inada conditions are sufficient to eliminate the possibility of reaching a state with infinite shadow prices through an *incremental* reversible process, but such results do not apply to the type of major shocks we associate with large rapid disruptions such as the total collapse of a fish stock or the end of a civilization through a total collapse of capital stocks.

2. How infinite shadow prices can turn up endogenously—why the last fish in the lake is worth infinity

Consider a general autonomous optimal control problem with infinite time horizon where the instantaneous utility is $F(u, x)$; the stock grows according to $\dot{x} = f(x, u)$, where $A = [0, a) \subseteq [0, \infty)$ is the feasible set of x values and initial conditions $x(0)$ are given.

Thus, we are examining the following problem:

$$\max_{u \in U \subseteq \mathbb{R}} \int_0^{+\infty} F(x, u) e^{-\rho t} \mathrm{d}t \qquad (1)$$
$$\text{s.t} \dot{x} = f(x, u), x(0) = x_0$$

Optimality conditions for problems like Eq. (1) may be found in Theorem 9.11.1 in the work of Sydsæter et al. (2005), where the Hamiltonian is assumed to be non-convex in x and u. The value of u that maximizes the Hamiltonian can be written $u(x, \mu)$, where μ is the co-state variable or shadow price on x. Optimality requires that optimal time paths for x and μ satisfy:

$$\dot{\mu} = \rho\mu - F'_x(x, u(x, \mu)) - \mu f'_x(u(x, \mu), x) \qquad (2)$$