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An axiomatization of difference-form contest success functions☆

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ABSTRACT

This paper presents an axiomatic characterization of difference-form contests, that is, contests where agents' probability of victory depends on the difference of their effective efforts. This axiomatization rests on a pairwise comparison axiom that relates the winning probabilities of any pair of participants to their winning probabilities in a contest between the two of them. The resulting difference-form contest success function overcomes some of the drawbacks of the widely-used ratio-form. Contrary to other difference-form functions, the family we charaterize here can be scale invariant and have a positive elasticity of augmentation. By clarifying the properties of this family of contest success functions, this axiomatization can help researchers to find the functional form better suited to their application of interest.

1. Introduction

Despite the relevance and ubiquity of contests in the real world, contest theory is often criticized for its great reliance on a particular construct: The Contest Success Function (Hirshleifer, 1989). This function maps the efforts made by contenders into their probability of attaining victory or, alternatively, their share of the contested prize. Critics argue that the contest success function (CSF henceforth) is too reduced form, too much of a black-box. For instance, the widely-used Tullock CSF (Tullock, 1967, 1980), under which success in the contest depends on relative efforts, might seem sensible. But there is no obvious reason why this functional form should govern most types of contests, from interstate wars to sport competitions.[1] Because of this, the predictions of contest theory might be seen as too reliant on very specific functional forms rather than on sound economic principles.

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[1] For excellent surveys of the contest literature see Corchón (2007) and Konrad (2009).

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This view is somewhat unfair for two reasons: Firstly, because there are other areas of Economics where very specific functional forms are often assumed. Secondly, because there is an active and fruitful strand of research which in the last few years has provided foundations to the most frequently employed CSFs.[2] This literature has even addressed the econometric estimation of these functions.[3] As a result of these efforts, economists have now at their disposal a growing menu of well-founded CSFs to choose from. The next natural question is which type of CSF is better suited to each specific application. A systematic study of the properties of the different families of CSFs can contribute to that aim.

One family of contests assumes that winning probabilities depend on the difference of contenders' efforts. These *difference-form* contests were introduced by Hirshleifer (1989, 1991) and explored later by Baik (1998) and Che and Gale (2000) for the case of two-player contests. Difference-form CSFs have been shown to emerge naturally in a number of settings. Gersbach and Haller (2009) show that a linear difference-form CSF is the result of intra-household bargaining when partners must decide how much time to devote to themselves or to their partner. Corchón and Dahm (2010) microfound a difference-form CSF as the result of a game where contenders are uncertain about the type of the contest designer; by interpreting the CSF as a share, they also show that the difference-form coincides with the claim-egalitarian bargaining solution. Corchón and Dahm (2011) obtain the difference-form as the result of a problem where the contest designer is unable to commit to a specific CSF once contenders have already exerted their efforts. Skaperdas and Vaidya (2012) derive a separable difference-form CSF in a Bayesian framework where contenders produce evidence stochastically in order to persuade an audience of the correctness of their respective views. Finally, Polishchuk and Tonis (2013) obtain a logarithmic difference-form CSF using a mechanism design approach when contestants have private information over their valuation of victory. In summary, it is fair to conclude that difference-form CSFs are well micro-founded. However, little is known about their properties and about how these differ from the properties of the more often used ratio-form CSFs, where winning probabilities are a function of the ratio of contenders' effective efforts.

The present paper offers an axiomatic characterization of difference-form CSFs. This axiomatization rests on a Pairwise Comparison axiom that describes the winning probabilities of any two participants in the contest as a function of their winning probabilities in the contest between the two of them. Under this axiom, if a contender has a zero winning probability in the grand contest, he/she can still have a positive probability of defeating another participant in a direct confrontation. This contrasts with the Consistency axiom employed in the characterizations of the ratio-form CSF. Under this axiom, a contender with no chance of winning the grand contest has no chance either of defeating another participant.

Our Theorem 1 shows that the Pairwise Comparison axiom, together with two other axioms already employed in the literature, characterize a separable difference-form CSF which generalizes the difference-form CSF introduced by Che and Gale (2000). This family of separable CSFs also encompasses as particular cases the ones micro-founded in the aforementioned literature as well as the ones employed by Levine and Smith (1995), Rohner (2006), Besley and Persson (2008, 2009) and Gartzke and Rohner (2011).

With our axiomatization, we help to clarify the properties of difference-form CSFs. The family we characterize is different from the logistic difference-form function introduced by Hirshleifer (1989, 1991) and later generalized by Baik (1998). Under the logistic CSF winning probabilities are proportional to contenders' exponential efforts. This functional form belongs to the ratio family, as it satisfies the consistency axiom and not our Pairwise Comparison axiom. We also show that contrary to the logistic CSF and to the Baik (1998) CSF, our difference-form CSF can be scale invariant, i.e. homogeneous of degree zero, and that it can have a positive elasticity of augmentation.[4]

This paper contributes to the axiomatic work pioneered by Skaperdas (1996) and Clark and Riis (1998). Later, Münster (2009) extended this characterization from individual to group contests. Arbatskaya and Mialon (2010) and Rai and Sarin (2009) axiomatized multi-investment contests, whilst Blavatskyy (2010) did the same for contests with ties. More recently, Hwang (2012) axiomatized the family of CSF with constant elasticity of augmentation, which encompasses the logistic and the ratio forms as particular cases. Lu and Wang, (2015) characterized success functions for contests producing strict rankings of players, whereas Vesperoni (2013) axiomatized an alternative success function producing rankings of any type. Finally, Bozbay and Vesperoni (2014) characterized a CSF for conflicts embedded in network architectures. Let us add that in our axiomatization we make connections with the income inequality literature. The literature on inequality measurement offers valuable insights on the properties of functional forms which we employ at several points of the text.[5]

2. Axiomatization

Let us start by considering a group of $K \geq 2$ individuals indexed by $k = 1, \ldots, K$. Denote the set of individuals by $\mathbb{K}$. These K agents are in competition. They are engaged in a contest which can have only one winner. In Section 4 we generalize our analysis to the case of group contests.

[2] These characterizations fall into four main categories: Axiomatic, stochastic, optimally-designed and microfounded (Jia et al., 2013).

[3] For a detailed discussion of the econometric issues involved in the estimation of CSFs see Jia and Skaperdas (2011) and Jia et al. (2013).

[4] A positive elasticity of augmentation (Hwang, 2012) implies that the difference between the winning probabilities of two contenders diminishes when their efforts increase whilst keeping their difference constant.

[5] In this same spirit, Chakravarty and Maharaj (2014) characterize a new family of individual contests success functions which satisfy properties akin to the intermediate inequality and ordinal consistency axioms employed in the income distribution literature.