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Comparison of algebraic and analytical approaches to the formulation of the statistical model-based reconstruction problem for X-ray computed tomography

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ABSTRACT

The main aim of this paper is to investigate properties of our originally formulated statistical model-based iterative approach applied to the image reconstruction from projections problem which are related to its conditioning, and, in this manner, to prove a superiority of this approach over ones recently used by other authors. The reconstruction algorithm based on this conception uses a maximum likelihood estimation with an objective adjusted to the probability distribution of measured signals obtained from an X-ray computed tomography system with parallel beam geometry. The analysis and experimental results presented here show that our analytical approach outperforms the referential algebraic methodology which is explored widely in the literature and exploited in various commercial implementations.

1. Introduction

This paper is closely related to a basic medical imaging technique which is called computed tomography (CT). In particular, it is concerned with the problem of formulating image reconstruction from projections algorithms, which are the most important things for the development of this technique. When considering the reconstruction methods used in this type of medical imaging, the highly destructive effects of the X-ray radiation used in CT must be taken into account. It is argued that this kind of radiation is harmful to the health of patients being examined because it can lead to many serious illnesses, and this therefore creates a barrier to the development of this X-ray medical imaging technique. This fact generates some problems concerning the marketing of the technique. It is for these reasons that research is being undertaken to decrease the dose of radiation absorbed by patients. So, among others, a new idea has emerged to do it by using an appropriately formulated reconstruction algorithm. This kind of approach allows us to improve image quality at high resolution and/or to decrease the dose of X-ray radiation absorbed by the patient. This idea is exemplified by statistical image reconstruction algorithms, which take into consideration the probabilistic conditions present in the measurement systems of CT scanners, so as to limit the influence of this noise on the images obtained from the measurements. To date, some commercial solutions of such systems have been developed, which perform reconstruction processing iteratively in order to decrease the noise in the images. Their practical usefulness has been confirmed by many papers published in radiological journals, for example [1]. The most interesting approach, called MBIR (model-based iterative reconstruction) by its authors, is presented in such papers as [2,3], where a statistical model of the measurement signals is derived in an analytical way, and, based on this, an iterative reconstruction algorithm is formulated. One can say in general that all the most significant existing reconstruction algorithms belong to two basic approaches, which take into account the methodology of the signal processing concepts used in them: those called analytical methods, and those assigned to the strategy called the series expansion methods (formulation of reconstruction problem takes in this case an algebraic form) [4,5]. We can guess that the implementation of this last methodology in CT Hounsfield's historical first apparatus [6] was because of a lack of an alternative at that time. In subsequent generations of CT systems only reconstruction algorithms based on analytical methodologies were used. This is understandable when we bear in mind the huge sizes of the matrices which occur in the algebraic reconstruction problem and the consequent calculation complexity of reconstruction method based on this methodology. The analytical methodology considerably simplifies the number of calculations necessary and is therefore more appealing. Afterwards, the approach with algebraically reconstruction problem formulation was considered for the design of statistical reconstruction algorithms (see e.g. [2,7–10]), because it allows for accurate modeling of the projections statistics and it helps to avoid most of the

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distortion caused by them. The practical realization of that application of the algebraic problem formulation presents some significant technical difficulties, in particular: in the case of algorithms for 3D spiral cone-beam scanners, the methodology forces simultaneous calculations for all voxels in the range of the reconstructed 3D image, making the reconstruction problem extremely complex [7,11,12]. Moreover, establishing the coefficients of the algebraic forward model is time-consuming. It is caused by a huge number of these coefficients in this model, and a fact that it is impossible to keep all of them in memory together.

It is possible to avoid the above-mentioned difficulties connected with using the algebraic methodology by using an analytical strategy for the reconstructed image processing. We have already shown how to formulate the analytical reconstruction problem using the ML method for parallel scanner geometry [13,14], for fan-beams [15], and finally we have proposed a scheme of reconstruction method for the spiral cone-beam scanner [16]. In comparison with methods based on algebraic methodology, this analytical statistical 3D reconstruction algorithm has some serious advantages. First of all, we can use the FFT which drastically decreases the time of an iteration during the reconstruction process.

In this paper, we present considerations involved with an approach to the statistical reconstruction problem, originally framed by us, called the analytical approximate reconstruction problem (see e.g. [14,15]). This discussion is concerned in particular with the fundamental problem of the conditioning of this new reconstruction formula. We will attempt to prove that our reconstruction method allows us to improve the quality of the images obtained after reconstruction. Moreover, it is a much less computationally demanding approach, which makes the reconstruction algorithms based on the principles described here very attractive from a practical point of view.

2. Formulation of the reconstruction problem

In this paper, we compare two different approaches to the reconstruction problem: the algebraic and the analytical schemes of reconstruction used in medical applications. In both cases, the same geometry of the X-ray measurement system is used. Let us begin then with the presentation of the details of the design of the measurement system depicted schematically in Fig. 1.

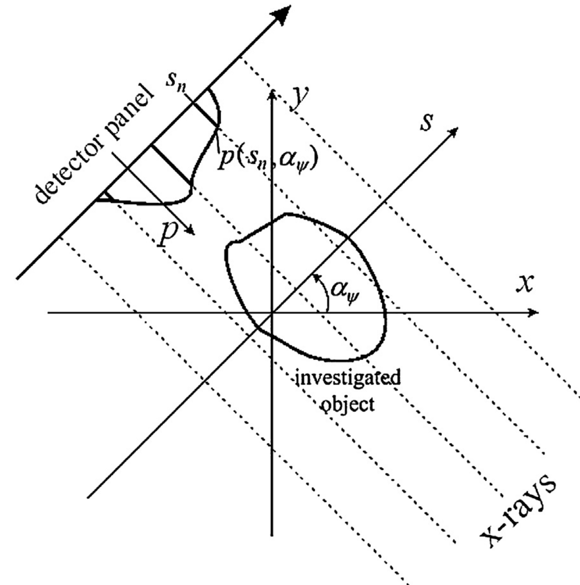

Fig. 1. Description of the parallel beam geometry of the X-ray scanner.

Practically, the reconstruction algorithm can only make use of projections p obtained at certain angles and measured only at particular points on the screen. In the case of the parallel beam scanner, a beam of X-rays reaches the individual detectors at points $n = -N/2, \ldots, N/2$, where N is an even number of virtual detectors from the projection obtained at angle α_ψ. Values $s_n = n \cdot \Delta_s$ denote the distances measured on the axis s between each parallel ray and the origin, where Δ_s is the sample interval on the detector panel. Parameters α_ψ denote discrete values of parallel projections taken at angles indexed by the variable ψ, where $\psi = 0, \ldots, \Psi - 1$, where Ψ is the number of projections, and every projection is carried out after a rotation by Δ_α. Summarizing, our reconstruction algorithm has available the projection values $p(s_n, \alpha_\psi)$, in the ranges: $n = -N/2, \ldots, N/2$; $\psi = 0, \ldots, \Psi - 1$. It is worth noting that we consider the discrete form of the image $\boldsymbol{\mu}(i, j)$, where $i = 1, \ldots, I, j = 1, \ldots, I$.

Algebraic reconstruction problem. First, we take into account what is for us a referential approach to the image reconstruction from projections problem, which belongs to a broader methodology, which makes use of finite series-expansion (see e.g. [5]). This reconstruction problem takes the form of a system of linear equations:

$$\mathbf{p} = \boldsymbol{\chi}\boldsymbol{\mu}, \tag{1}$$

where $\mathbf{p} = [p_k]$ is the projection vector with $k = 1, \ldots, N \cdot \Psi$; $\boldsymbol{\chi}$ is the matrix of coefficients $\boldsymbol{\chi}_{l\psi,ij}$ with dimensions $N \cdot \Psi \times I^2$; $\boldsymbol{\mu}$ is a vector formed by the successive placing of the rows $i = 1, \ldots, I$ of array $\boldsymbol{\mu}(i, j)$ into the vector $\boldsymbol{\mu}$. In general, the elements $\boldsymbol{\chi}_{l\psi,ij}$ can be interpreted physically as the contribution of a given image block with parameters (i, j) to the formation of the projection value identified by the pair (n, ψ), measured at the screen. For instance, these coefficients may be determined using a distance-driven method [11]. Because of the algebraic nature of the linear equation system (1), let us call the approach presented here as algebraic.

According to statistical considerations (see e.g. [2] or [3]) the following maximum likelihood estimate of the reconstructed image may be computed according to the following relation:

$$\boldsymbol{\mu}_0 = \arg\min_{\boldsymbol{\mu}} \left(\frac{1}{2} (\mathbf{p} - \boldsymbol{\chi}\boldsymbol{\mu})^{\mathrm{T}} \mathbf{D} (\mathbf{p} - \boldsymbol{\chi}\boldsymbol{\mu}) \right), \tag{2}$$

where $\mathbf{D}$ is a diagonal matrix:

$$\mathbf{D} = \begin{bmatrix} d_1 & 0 & \cdots & 0 \\ 0 & d_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & d_{N \cdot \Psi} \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_{N \cdot \Psi} \end{bmatrix}. \tag{3}$$

In addition, it is assumed [17] that:

$$d_k \cong \frac{1}{\sigma_{p_k}}, \tag{4}$$

where σ_{p_k} are the variances of the projection measurements p_k.

Formula (2) is indeed a Weighted Least Squares (WLS), and can be solved using for example any gradient descent method.

Analytical reconstruction problem. The analytical approach to the image reconstruction from projections problem considered here is based on the 2D analytical approximate reconstruction problem for parallel geometry of scanner originally formulated by us (see e.g. [14,18]), and can be presented as a shift-invariant system:

$$\tilde{\boldsymbol{\mu}}(i, j) \cong \sum_{\bar{i}} \sum_{\bar{j}} \boldsymbol{\mu}(\bar{i}, \bar{j}) \cdot h_{\Delta i, \Delta j}, \tag{5}$$