Cognitive Development 44 (2017) 1–11

Contents lists available at ScienceDirect

Cognitive Development

journal homepage: www.elsevier.com/locate/cogdev

Relations between patterning skill and differing aspects of early mathematics knowledge☆

Emily R. Fyfe[a,*], Julia L. Evans[b], Lauren Eisenband Matz[c,1], Kayla M. Hunt[d,2], Martha W. Alibali[d]

[a] Wisconsin Center for Education Research, University of Wisconsin-Madison, United States
[b] School of Behavioral and Brain Sciences, University of Texas, Dallas, United States
[c] Department of Communication Sciences and Disorders, University of Wisconsin-Madison, United States
[d] Department of Psychology, University of Wisconsin-Madison, United States

ARTICLE INFO

Keywords:
Pattern knowledge
Mathematics
Calculation
Concepts
Working memory

ABSTRACT

Patterns are often considered central to early mathematics learning; yet, the empirical evidence linking early pattern knowledge to mathematics performance is sparse. In the current study, 36 children ranging in age from 5 to 13 years old (M = 9.1 years) completed a pattern extension task with three pattern types that varied in difficulty. They also completed three math tasks that tapped calculation skill and knowledge of concepts. Children were successful on the pattern extension task, though older children fared better than younger children, potentially due in part to their explanations that considered both dimensions of the pattern (shape and size). Importantly, success on the pattern extension task was related to mathematics performance. After controlling for age and verbal working memory, patterning skill predicted calculation skill; however, patterning skill was not associated with knowledge of concepts. Results suggest that patterning may play a key role in the development of some aspects of early mathematics knowledge.

1. Introduction

Patterns are ubiquitous in early learning environments, particularly in mathematics. Whether creating an alternating sequence of blue and red cubes or determining the next number in a function table, children often must make predictions based on the rule governing a pattern. Some have argued that “patterns are what mathematics is all about” and have defined mathematics as the science of patterns (Steen, 1988, p. 616). Further, recent empirical research has provided both correlational and causal evidence relating patterning skills to mathematics achievement (Kidd et al., 2013; Kidd et al., 2014; Papic, Mulligan, & Mitchelmore, 2011; Rittle-Johnson, Fyfe, Hofer, & Farran, 2016). However, little is known about how patterning skill relates to different types of mathematics knowledge − for example, calculation skill versus knowledge of specific math concepts. The goals of the current study

☆ Support for this research was provided by grant R0105263 from the National Institute on Deafness and Other Communication Disorders (Julia L. Evans, PI) and by training grant award R305B130007 from the Institute of Education Sciences, U.S. Department of Education. The views expressed are those of the authors and do not represent the views of the U.S. Department of Education.

* Corresponding address. Present address: Department of Psychological and Brain Sciences, Indiana University, 1101 E. 10th Street, Bloomington IN 47405, United States.

E-mail address: efyfe@indiana.edu (E.R. Fyfe).

[1] Present address: Lab School of Washington, United States.

[2] Present address: University of Rochester Medical Center, Department of Psychiatry, United States.

http://dx.doi.org/10.1016/j.cogdev.2017.07.003
Received 11 May 2016; Received in revised form 21 April 2017; Accepted 25 July 2017
Available online 05 August 2017

were (1) to document children's performance on a novel pattern extension task, and (2) to examine relations between patterning skill and performance on three mathematics tasks that tap both calculation skill and knowledge of concepts.

One of the first types of patterns that children engage with are repeating patterns − linear patterns with a unit that repeats (e.g., ABABAB; National Association for the Education of Young Children, 2014; National Council of Teachers of Mathematics, 2006). The unit usually contains two or three elements (e.g., AB, ABB, ABC) that vary on a key dimension, such as color (e.g., red-blue-red-blue-red-blue), shape (e.g., square-circle-square-circle-square-circle), or size (e.g., big-small-big-small-big-small). Empirical research has provided correlational and causal evidence relating repeating patterning knowledge to mathematics achievement. For example, in a longitudinal sample, repeating pattern skill at the end of pre-k, kindergarten, and first grade was a unique predictor of mathematics achievement in fifth grade, even after controlling for other mathematics, reading, cognitive, and demographic measures (Rittle-Johnson et al., 2016; see also Fyfe & Rittle-Johnson, 2016).

Several intervention studies have documented relations between repeating pattern knowledge and mathematics performance. In a non-experimental, classroom-based study involving instruction on repeating patterns, Warren and Cooper (2007) examined children's discussions of target patterning and ratio tasks. They argued that repeating patterns served as an "effective bridge" (p. 14) to understanding ratios (e.g., if the ratio of juice to water in jug 1 is 2:4 and the ratio of juice to water in jug 2 is 4:8, which jug has the stronger juice?). In a quasi-experimental study, Papic et al. (2011) examined the effectiveness of a six-month repeating pattern intervention. Children in a preschool that used the intervention exhibited better skills with growing patterns at the end of the year than children in a preschool that did not. Further, children who received the intervention scored higher on a standardized number assessment at the end of kindergarten than children who did not.

Two randomized controlled trials provide causal evidence that instruction on repeating patterns supports mathematics knowledge. In two separate studies, Kidd and colleagues (Kidd et al., 2013; Kidd et al., 2014) randomly assigned struggling first-grade students to receive supplemental instruction in patterning, mathematics, reading, or social studies. The patterning instruction included repeating patterns and other pattern types (e.g., symmetrical patterns, rotation patterns) and occurred three times a week over a six-month period. The patterning intervention resulted in comparable or better performance on standardized mathematics assessments, relative to the other interventions.

Clearly, an emerging body of research suggests that children's repeating pattern knowledge may play a pivotal role in their mathematics performance. However, the number of studies documenting this relation remains small. More importantly, the few studies that have documented this pattern-math relation have relied primarily on global measures of mathematics achievement. The one exception by Warren and Cooper (2007) relied on qualitative descriptions of classroom episodes. It is possible that patterning skill may be differentially related to different aspects of math knowledge. For example, a distinction is often made between children's ability to calculate an answer and the conceptual knowledge that underpins correct strategies (e.g., Crooks & Alibali, 2014; Rittle-Johnson & Alibali, 1999; Siegler, 2000). To date, no studies have investigated whether children's pattern skill is associated with different aspects of mathematics knowledge in similar or different ways.

The primary goal of the current study was to examine the relations between children's patterning skill and different aspects of their mathematics performance. We selected three math tasks that allowed us to examine children's calculation skill and their knowledge of important mathematical concepts. First, we included standard arithmetic problems in an operations-equals-answer format (e.g., 2 + 4 + 5 + 2 = _). These problems are routine for young children, and they can be used to tap calculation skill. Second, we included mathematical equivalence problems, which contain operations on both sides of the equal sign (e.g., 3 + 4 + 6 = 3 + _). To solve these problems correctly, children must realize that the equal sign indicates an equivalence relation − that is, the amounts on the two sides must be the same. Thus, these problems tap children's understanding of mathematical equivalence, a key concept that children acquire during their elementary school years, but that is challenging for many children in the US and Canada (e.g., Falkner, Levi, & Carpenter, 1999; Kieran, 1981; McNeil & Alibali, 2005). Third, we included inversion problems, in which the same value is added and subtracted (e.g., 4 + 7 − 7 = _). Children can solve these problems correctly either by performing each operation (e.g., "4 plus 7 is 11, and 11 minus 7 is 4") or by recognizing the inversion principle, which holds that adding and subtracting the same quantity results in no net change (e.g., "the 7s cancel so it keeps the 4 the same"). Thus, these problems tap children's understanding of mathematical inversion, another key concept that children typically acquire during their elementary school years, but that is challenging for many children (Gilmore & Bryant, 2008; Robinson, Ninowski, & Gray, 2006). Both equivalence and inversion problems have been widely used in the literature on the development of conceptual knowledge (see Crooks & Alibali, 2014, for a review).

We examined children's performance on these three math tasks as it related to their repeating pattern knowledge. Research has documented children's performance on a variety of repeating patterning tasks − including copying a model pattern with the same or different materials and identifying the unit that repeats (e.g., Clements, Sarama, & Liu, 2008; Fyfe, McNeil, & Rittle-Johnson, 2015; Mulligan & Mitchelmore, 2009; Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013; Rittle-Johnson, Fyfe, Loehr, & Miller, 2015; Warren & Cooper, 2006). Here, we focused on a pattern extension task, which involves continuing a model pattern (e.g., "what comes next?"). Pattern extension is a common and popular patterning task for young children (Economopoulos, 1998; Rittle-Johnson et al., 2015). Further, it is accessible at an early age, but with substantial variability and room to improve (e.g., Aubrey, 1993; Starkey et al., 2004; Rittle-Johnson et al., 2013). Thus, it was well suited to investigate pattern knowledge in early childhood. In previous work, item difficulty has often been defined by the pattern task (e.g., copying versus extending). We opted instead to use a single pattern task (i.e., pattern extension) and to vary the difficulty of the type of patterns used within the task. Thus, a secondary goal of the current study was to document children's performance on this novel pattern extension task. To do so, we examine both children's overall level of success and the nature of the strategies they used in extending different types of patterns.

In the current study, children saw patterns with elements that varied in shape (circle or square) and size (big or small), and had to