Cognitive Development 41 (2017) 73–84

Contents lists available at ScienceDirect

Cognitive Development

Estimation as analogy-making: Evidence that preschoolers' analogical reasoning ability predicts their numerical estimation

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ARTICLE INFO

Article history:
Received 8 October 2015
Received in revised form 11 October 2016
Accepted 1 December 2016
Available online 27 December 2016

Keywords:
Numerical cognition
Estimation
Analogy

ABSTRACT

All humans and many animals can represent approximate quantities of perceptual objects nonlinguistically by using the Approximate Number System (Dehaene, 1997/2011). Early in life, children in numerate societies also learn to describe this system using number words. How do linguistic representations of number become related to nonlinguistic representations of number? We hypothesize that the analogical process of structure mapping (Gentner, 1983) helps children to form mappings between the linguistic and nonlinguistic number systems on the basis of structural similarities between the two systems. To test this, we tested and analyzed 47 four-and-five year olds' performance on estimation and analogy tasks. We found that analogical reasoning ability uniquely predicted several components of estimation performance, even when controlling for other domain-general cognitive skills. This provides strong evidence that analogical processes are uniquely related to the development of early estimation.

We rarely have the time or inclination to count all of the quantities that we might want to keep track of (e.g., coins in your car, the number of chips you just ate); fortunately, we can estimate! The ability to provide numerical estimates – that is, to provide a linguistic numerical label for a numerical quantity – develops in the first few years of life (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Le Corre & Carey, 2007; Lipton & Spelke, 2005), and predicts math success (Booth & Siegler, 2006, 2008; Moore & Ashcraft, 2015; Siegler & Booth, 2004). Much of the existing research on numerical estimation has thus far focused on understanding what estimation tasks can tell us about the development of numerical competence. However, while estimation undoubtedly requires numerical knowledge, because estimation requires connecting the linguistic and nonlinguistic number systems to one another, it serves as an excellent test case for understanding the more general question of how language becomes related to perception during development (Sullivan & Barner, 2012). In the present study, we ask whether one domain-general cognitive ability – the ability to relate disparate domains to one another analogically – supports early estimation.

When we estimate, we label a perceptually available quantity (e.g., a set of dots; a series of beeps) with a number word – in other words, we describe our nonlinguistic numerical representations by using linguistic representations of number. One nonlinguistic number system is the evolutionarily ancient Approximate Number System (ANS), which allows us to roughly quantify e.g., the number of objects in a set (albeit with systematic error; it cannot represent quantities like "exactly

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http://dx.doi.org/10.1016/j.cogdev.2016.12.004
0885-2014/

56"; Dehaene, 1997/2011). In contrast, our linguistic number system precisely represents numerical quantity in a culturally-mediated and uniquely human format (e.g., via number words or numerals). Because the linguistic and nonlinguistic number systems differ radically in representational format, recent work has focused on understanding the mappings between these systems, and has done so by studying the development of estimation ability (Sullivan & Barner, 2012, 2014a).

How do the linguistic and nonlinguistic number systems become mapped onto one another? While it is theoretically possible that the linguistic and nonlinguistic number systems become related to one another via associative learning (e.g., by associating words like *eight* with ANS representations of 'about 8 things'; Lipton & Spelke, 2005), such a learning process would require substantial experience with specific word-magnitude pairings (which four-year-olds, who are learning these mappings, almost certainly lack), and has been empirically demonstrated to play only a minimal role in connecting number words to the ANS (Lyons, Ansari, & Beilock, 2012; Sullivan & Barner, 2012, 2014a). One alternative possibility is that children recruit structure mapping (Gentner, 1983; Gentner & Markman, 1997; Gentner & Namy, 2006), an analogical process, to form holistic mappings between their linguistic and nonlinguistic number systems, based on the structural similarities between these two systems (Sullivan & Barner, 2012, 2014a). For example, a child might compute an analogy based on the shared ordinal structure of the linguistic and nonlinguistic number systems: they might notice that the word *sixty* comes after the word *thirty* in the count list, just like sets containing 60 items appear more numerous than sets containing 30 items (in standard analogical notation, this might be presented as *sixty* : *thirty* :: 60 : 30). This, in turn would lead to the inference that *sixty* should be mapped onto larger sets than *thirty* is (see also, Lyons & Beilock, 2011). Or, a child might compute an analogy based on the relative-distance structural relations encoded within the linguistic and nonlinguistic number systems: they might notice that the word *sixty* comes twice as far along in the count list as the word thirty, and therefore infer that *sixty* should be mapped onto a quantity that is twice as large as that referred to by *thirty*. In this sense, estimation could be supported by analogical inferences about the relation between the structures of the linguistic and nonlinguistic number systems (for additional information about how analogy might support still other components of estimation, see Thompson & Opfer, 2010).

There is substantial evidence consistent with the view that analogical reasoning supports the connection between linguistic and non-linguistic representations of number during estimation. First, when participants are given misleading feedback (e.g., by being shown that a particular quantity is 30, or by being told that the largest quantity that they will encounter during an estimation task is 750), they recalibrate their estimates for all but the smallest numbers (Izard & Dehaene, 2008; Sullivan & Barner, 2012, 2014a). In other words, most number word mappings are mutually constraining – when the mapping for one number is altered, estimates for most other numbers are affected. These calibration effects should emerge if participants have a holistic, structure mapping between their linguistic and nonlinguistic number systems, and not if item-by-item associations underlie number word mappings (Sullivan & Barner, 2012, 2014a,b). Second, even when participants are not given explicitly misleading feedback, recent work has shown that their estimates are mutually constraining, such that estimation error (e.g., over- or under-estimating) carries forward from trial to trial (Sullivan, Juhasz, Slattery, & Barth, 2011; Vul, Barner, & Sullivan, 2013). Again, this suggests that participants use structure mapping to relate the mapping for the current trial to mappings deployed on earlier trials, such that mappings are internally consistent.

Finally, a signature of the use of analogy during estimation is that estimates are internally consistent, even when they are inaccurate. Internally consistent estimates emerge when estimates (and therefore mappings between number words and the ANS) are not mutually contradictory. In other words, in order for estimates to be internally consistent, they must make sense with respect to one another. For example, estimates that are in the correct order relative to one another (e.g., larger number words are used to label larger quantities), or that are linearly related to one another, are internally consistent, even if they happen to be inaccurate. Children's early estimates exhibit two markers of an internally consistent structure mapping. First, children are able to provide internally consistent estimates (e.g., ordinal estimates that are in the correct order relative to previous trials) long before they provide accurate estimates, and even for quantities outside the child's stable count range (Barth, Starr, & Sullivan, 2009; Sullivan & Barner, 2014a,b). Second, children's estimates are often predictively related to one another (e.g., are non-random), even when they are wildly inaccurate (Barth et al., 2009; Berteletti et al., 2010; Booth & Siegler, 2006, 2008). This suggests that the earliest mappings between the linguistic and nonlinguistic number systems are unlikely to be based on veridical pairings of number words to magnitudes (this view would predict that only accurate estimates should be internally consistent, and that estimates for unmapped magnitudes should be random), and instead are based in the relation between the structures of the two number systems (Thompson & Opfer, 2010). While these studies are consistent with the view that analogy underlies estimation, previous work has not directly tested the relation between estimation and domain-general analogical reasoning skill.

If estimation requires children to analogically relate their linguistic and nonlinguistic number systems to one another, then it is likely that children who are most able to compute analogies will also be most likely to recruit structure mapping when estimating. For this reason, we tested 4- and 5-year-old children's analogical reasoning skill, and asked whether it uniquely predicted the elements of estimation that have been argued to be analogical in nature (i.e. the effect of calibration, ordinality scores, and linear r^2), while controlling for a host of other non-analogical factors (age, counting ability, memory, and Raven's Progressive Matrices).