Cognitive Development 43 (2017) 14–24

Contents lists available at ScienceDirect

Cognitive Development

Procedural complexity underlies the efficiency advantage in abacus-based arithmetic development

Chris Donlan*, Claire Wu

Department of Language and Cognition, University College London, United Kingdom

ARTICLE INFO

Article history:
Received 7 May 2016
Received in revised form 11 January 2017
Accepted 7 February 2017
Available online 23 February 2017

Keywords:
Abacus
Complementary number
Calculation

ABSTRACT

The abacus is a counting frame designed to facilitate the process of arithmetic calculation. This paper focusses particularly on the Japanese 'soroban' abacus, examining the role of complementary numbers in its operation. We propose that the combination of quinary (base five) and decimal (base ten) systems, and the consequent use of complementary numbers (CN) in calculation, are transferred from physical to mental procedures. The more CN steps involved in a calculation, the more time it should take for mental abacus users to complete. This hypothesis was tested in Experiment 1: 146 Taiwanese abacus users aged 3–15, identified as either abacus learners (n = 126) or abacus experts (n = 20), were given parallel sets of serial addition and subtraction problems, matched on conventional metrics of difficulty level, but differing in the number of CN steps entailed in abacus calculation. An effect of CN was found whereby response times were significantly greater for the condition involving double CN steps. Experts were significantly faster than learners, but the effect of CN did not differ between groups. In Experiment 2, in order to test the specificity of the CN effect, a group of British children (n = 20), with no experience of the abacus, was given the same mental calculation tasks. They achieved accuracy levels equivalent to the abacus experts, but their calculation speed was very much slower, and they showed no evidence of the CN effect.

Our findings demonstrate the importance of complementary numbers in children's mental abacus calculation. We found the CN effect to be equally strong in expert and non-expert users, and confirmed the specificity of the effect by showing it to be absent in participants not exposed to abacus use. We argue that the use of complementary numbers in mental abacus calculation provides learners, from the outset, with exposure to a dual-base system which, while procedurally complex, affords a representational advantage not enjoyed by learners working within the exclusively decimal system of Arabic numerals.

1. General introduction

The development of mental representations underlying the performance of arithmetic is a research focus of high theoretical and practical importance. Effort has been devoted to establishing number as a primitive domain of cognition (Xu and Spelke, 2000), and to exploring the relation between early non-symbolic number representations and later school attainment in arithmetic (Halberda, Mazzocco, & Feigenson, 2008). At the same time, evidence is accumulating of the early influence

* Corresponding author at: Department of Language and Cognition, University College London, 2, Wakefield St., London WC1N 1PF, United Kingdom.
E-mail address: c.donlan@ucl.ac.uk (C. Donlan).

http://dx.doi.org/10.1016/j.cogdev.2017.02.002

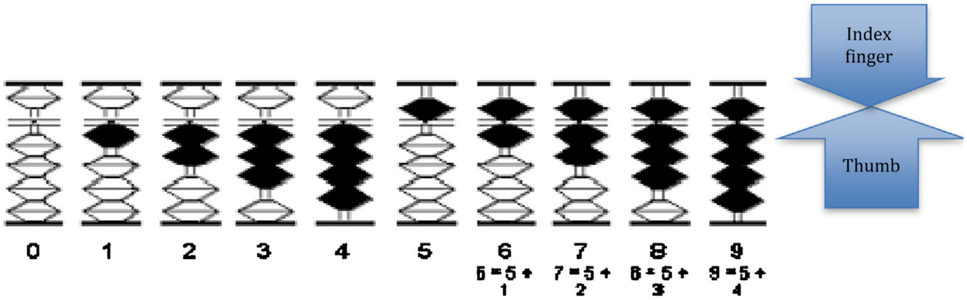

Fig. 1. A soroban abacus representing "one hundred and twenty three million, four hundred and fifty-six thousand, seven hundred and eighty-nine".

of culturally determined symbol systems on children's arithmetic (Nunes, 2003; Almoammer et al., 2013; Goebel, Watson, Lervag, & Hulme, 2014). Little psychological research has so far been devoted to the use of the abacus counting frame as a vehicle for arithmetic learning. A recent study has suggested that the success of the abacus as a cognitive tool may be due to its close adaptation to parameters of the human memory system (Frank and Barner 2011). The current study explores this issue from a developmental standpoint. Based on observation of early instruction procedures we propose that the multiple levels of representation which the abacus entails are advantageous not only insofar as they are adapted to the constraints of human memory, but also insofar as they afford early procedural learning of key conceptual relations (complementary numbers). In order to substantiate this claim we first provide an account of the nature of the Soroban Abacus and its educational properties.

1.1. Design and operation of the Soroban Abacus

The abacus, of which there are many types across the world, is a counting frame containing sets of moveable beads, mounted on rods. The particular model discussed throughout this paper is the Japanese soroban-abacus (see Fig. 1, below). The abacus is made of two parts (the upper part, which contains the "heaven beads" and the lower part, which contains the "earth beads") separated by a beam. The heaven bead is composite unit (five), the earth bead is a simple unit. A single heaven bead and four earth beads are combined on each rod. Each rod represents a digit within a base-ten (decimal) numeral (Kojima, 1954). Pushing one earth bead up towards the beam represents the value one, the same logic applying from one to four. Pushing the heaven bead downward to the beam represents the value five. Abacus representations 1–9 are shown within a nine digit numeral in Fig. 1, below. Particularly noteworthy is the complex nature of abacus representation. At once it offers (a) direct perceptual representation of the numerosity of units, (b) composites (heaven beads) providing a quinary base, and (c) the frame of rods providing decimal place value. The latter, and only the latter, is shared with the Arabic numeral system.

The primary purpose of the abacus is to perform calculation tasks mechanically, thereby minimizing mental workload, by a process that Kojima (1954) called 'mechanicalization'. This is achieved by users familiarizing themselves with sets of complementary numbers (CN), particularly those number pairs which sum to 5 (the value of the heaven bead) and 10 (the base unit on which the abacus framework is constructed).

Calculation using the soroban abacus requires knowledge of two sets of CN, based on the heaven bead (value 5) and the base system on which the abacus frame is constructed (value 10). The user must know that $1+4=5$, $2+3=5$, $3+2=5$, $4+1=5$, and that $1+9=10$, $2+8=10$, $3+7=10$, $4+6=10$, $5+5=10$, $6+4=10$, $7+3=10$, $8+2=10$ and $9+1=10$. The CN are taught to young children as sets of "friends", e.g., referring to the heaven bead, 3 has the 'friend' 2, and vice versa. Before a calculation can be made, all rods must be set to zero. Some single rod additions and subtractions can be made without using CN. Fig. 2 (below) shows step-by-step examples, showing the state of the abacus at each stage.

Where a particular calculation cannot be directly represented by simple movement of beads, the user must now think in terms of CN. The learning opportunity afforded by the use of CN is demonstrated in the operations of simple addition and subtraction, which are taught together. The inverse relation between these operations (if $a+b=c$, then $c-b=a$) is procedurally represented in the abacus calculation.

An example is given in Fig. 3, above. In each case the same set of CN of 5 is employed in both addition and subtraction. This use of complementary numbers, providing procedural representation of the principle of inversion, is practiced from the outset of arithmetic instruction (as early as age 3 or 4). These procedures are complex compared to those typically employed in early arithmetic learning using Arabic numerals. Instruction is largely based on counting (Siegler 1987; Donlan, Cowan, Newton, & Lloyd, 2007), often implemented using a simple linear representation or 'number line', and always operating within a strictly decimal system. Under these conditions systematic exposure to the inversion principle is likely to occur at a much later stage e.g. 7–8 years (Canobi, 2009). Iterative patterns in the development of conceptual and procedural knowledge have been widely observed. Transfer from practising procedures to understanding concepts has proved particularly effective where the relation is made explicit (Rittle-Johnson & Schneider, 2015). The multiple levels of representation of the abacus would seem to lend themselves well to this form of transfer.