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The unique and shared contributions of arithmetic operation understanding and numerical magnitude representation to children's mathematics achievement

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ABSTRACT

The current study examined the unique and shared contributions of arithmetic operation understanding and numerical magnitude representation to children's mathematics achievement. A sample of 124 fourth graders was tested on their arithmetic operation understanding (as reflected by their understanding of arithmetic principles and the knowledge about the application of arithmetic operations) and their precision of rational number magnitude representation. They were also tested on their mathematics achievement and arithmetic computation performance as well as the potential confounding factors. The findings suggested that both arithmetic operation understanding and numerical magnitude representation uniquely predicted children's mathematics achievement. The findings highlight the significance of arithmetic operation understanding in mathematics learning.

Introduction

Mathematics is a compulsory subject for elementary school students. Mastery of mathematical skills is related to better educational attainment, financial status, and psychological well-being (Parsons & Bynner, 2005; Ritchie & Bates, 2013). An increasing number of studies have been conducted

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to examine how children learn these important skills (Geary, 2010). Solving mathematical problems, even the simplest ones such as 3 + 2 = __, requires the understanding of both numerical magnitude (i.e., how much do 3 and 2 represent?) and arithmetic operations (i.e., what is the meaning of +?). Inadequate understanding in either aspect would make mathematics learning difficult. For instance, whereas an inadequate understanding of numerical magnitude may hinder children from doing arithmetic computation (Wong, Ho, & Tang, 2014), children will probably get stuck on mathematical problem solving if they do not know which arithmetic operations are needed for calculating the area of a triangle in geometry, and for finding out the averages in statistics, even though they may have good understanding about numerical magnitude. Although it seems obvious that understanding of both numerical magnitude and arithmetic operations is essential to mathematics learning, the unique contributions of the two constructs remain largely unexplored. The current study, therefore, was conducted to fill this important gap.

Numerical magnitude representation

Numerical magnitude representation is probably the most extensively researched topic within the field of mathematical cognition. The major underlying assumption behind the huge body of research is that the representation of numerical magnitude serves as the basis on which our mathematics skills can be developed (e.g., Dehaene, 2001; Feigenson, Dehaene, & Spelke, 2004; Geary, 2013; Siegler, 2016). In the integrated theory of numerical development, Siegler and his colleagues (Siegler, 2016; Siegler, Thompson, & Schneider, 2011) have proposed four major developments in numerical magnitude knowledge and how these developments are related to our mathematics skills. The first development involves the refinement of nonsymbolic numerical magnitude representation. Humans, like many other animals, are equipped with an innate ability to represent nonsymbolic numerosity. Such ability allows us to discriminate between numerosities that differ by a certain ratio. For example, 6-month-old infants can discriminate between arrays of 8 and 16 dots but not between arrays of 8 and 12 dots (Xu & Spelke, 2000). This ability, also known as the number acuity, improves with age. Whereas kindergarteners are able to reliably discriminate arrays of 9 versus 12 dots, the ratio is sharpened to approximately 9:10 during adulthood (Halberda & Feigenson, 2008; Piazza et al., 2010). Because this approximate sense of number does not allow us to work with precise quantities, we need to develop a symbolic number system and associate the symbolic numbers with this nonsymbolic numerical representation (Dehaene, 2005). This is the second major development of numerical magnitude knowledge. Children acquire the meaning of the number symbols through making such associations. After they have acquired the meanings of the small numbers, they bootstrap their understanding about these small numbers to the larger numbers. For example, by analogizing 1500 to 10,000 as 15 to 100, children can better understand the magnitude of large numbers based on their representation of magnitudes of small numbers (Thompson & Opfer, 2010). In the final step of numerical magnitude development, children learn about rational numbers. They need to realize that although many of the rules that apply to whole numbers (e.g., each whole number has a unique predecessor and successor and is represented by a unique symbol) do not apply to rational numbers, all rational numbers can be represented as magnitudes on the mental number line. With the interference of whole number knowledge (also known as the whole number bias; Ni & Zhou, 2005), mastering the magnitude of rational numbers seems to be particularly difficult. Even for tasks as simple as fraction comparison, the performance of undergraduates is far from perfect (DeWolf, Grounds, Bassok, & Holyoak, 2014). It seems that not every adult reaches this final stage of numerical development.

Understanding of arithmetic operations

Compared with numerical magnitude representation, we have relatively little understanding about how children understand arithmetic operations. The term *arithmetic operation understanding* refers to the sense about the four arithmetic operations, such as when they should be used and the implicit regularities behind them, instead of the ability to perform arithmetic computations. In other words, it refers to whether individuals know they should use multiplication to find out the total costs required