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Approximate number sense correlates with math performance in gifted adolescents

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ABSTRACT

Nonhuman animals, human infants, and human adults all share an Approximate Number System (ANS) that allows them to imprecisely represent number without counting. Among humans, people differ in the precision of their ANS representations, and these individual differences have been shown to correlate with symbolic mathematics performance in both children and adults. For example, children with specific math impairment (dyscalculia) have notably poor ANS precision. However, it remains unknown whether ANS precision contributes to individual differences only in populations of people with lower or average mathematical abilities, or whether this link also is present in people who excel in math. Here we tested non-symbolic numerical approximation in 13- to 16-year old gifted children enrolled in a program for talented adolescents (the Center for Talented Youth). We found that in this high achieving population, ANS precision significantly correlated with performance on the symbolic math portion of two common standardized tests (SAT and ACT) that typically are administered to much older students. This relationship was robust even when controlling for age, verbal performance, and reaction times in the approximate number task. These results suggest that the Approximate Number System is linked to symbolic math performance even at the top levels of math performance.

1. Introduction

Mathematical thinking permeates modern human life and supports a wide range of activities, from counting change after a purchase to formulating theoretical proofs. Because we rely on mathematics in so many ways, it is not surprising that math competence predicts a variety of long-term outcomes such as job attainment, salary, financial literacy, and personal debt (Dougherty, 2003; Gerardi, Goette, & Meier, 2013; Parsons & Bynner, 2005; Rivera-Batiz, 1992; Roszkowski, Glatzer, & Lombardo, 2015). Individual differences in math ability can be seen starting early in life: whereas some children consistently have difficulty mastering math procedures and concepts (Butterworth, Varma, & Laurillard, 2011; Geary, 2004), other children demonstrate advanced mathematical performance early on (Brody & Mills, 2005).

Many factors have been shown to predict children's math achievement, including family income (e.g., Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006), quality of early childcare (Dearing, McCartney, & Taylor, 2009), teachers' math knowledge (Hill, Rowan, & Ball, 2005), quantity of teachers' math input (Klibanoff et al., 2006), and domain general abilities such as working memory, attention, and executive control (e.g., Clark, Pritchard, & Woodward, 2010; Geary, 2011; Mazzocco & Kover, 2007; Welsh, Nix, Blair, Bierman, and Nelson (2010). But in addition to these factors, an emerging body of research has found that an evolutionarily ancient, non-symbolic sense of quantity is linked with math performance in children and adults. This number sense is seen in human infants starting in the first few days of life (Izard, Sann, Spelke, & Streri, 2009) as well as in a variety of non-human animals including monkeys, rats, chicks, and fish (for review see Agrillo, Piffer, Bisazza, & Butterworth, 2012; Brannon & Merritt, 2011; Feigenson, Dehaene, & Spelke, 2004). While none of these non-verbal creatures represents exact integer quantities, they all can represent quantity in an approximate way. The Approximate Number System (ANS) representations that underlie their performance have been described "noisy," with the amount of noise or uncertainty in the numerical representations scaling with target size (i.e., greater uncertainty for larger values). As a result, observers' ability to numerically discriminate two arrays using the ANS depends on the arrays' ratio rather than their absolute difference (e.g., Halberda & Odic, 2014; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Whalen, Gallistel, & Gelman, 1999). In humans, the noisiness of ANS representations decreases over development, starting in infancy and continuing into adulthood (Halberda & Feigenson, 2008; Lipton & Spelke, 2003; Xu, Spelke, & Goddard, 2005), with humans' highest precision attained at around 30 years of age (Halberda, Ly, Wilmer, Naiman, & Germine,

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2012).

Noisy ANS representations are fundamentally different from the precise integer representations that play a key role in human mathematical reasoning. For example, discriminating the nearby quantities 49 and 50 using the ANS is difficult (and would yield performance that is only just above chance in humans (Halberda, 2016)), but discriminating 49 from 50 using integer representations is easy. Whereas approximate number representations are used by humans starting in infancy, integer representations are not exhibited until around age four, after the verbal counting procedure is acquired (Carey, 2009; Wynn, 1992). Whereas no human culture has been documented to lack approximate number representations, adult humans in cultures that lack number words appear to lack integer representations (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). And whereas a wide range of animal species represents approximate number, no non-human species has shown evidence of representing integers.[1]

Yet despite these differences between approximate number representations and the integer representations on which so much of formal mathematics depends, evidence suggests a link between the two. In particular, individual differences in ANS precision have been found to correlate with symbolic math performance in multiple age groups (Chen & Li, 2014; Feigenson, Libertus, & Halberda, 2013 for a review). In such studies, ANS precision typically is measured by showing observers two stimulus arrays (e.g., dot displays) and testing their ability to reliably judge which is more numerous. Although this non-verbal, non-symbolic task is trivially easy when the ratio between the arrays is large (e.g., 20 dots versus 40 dots), it becomes much harder when the ratio is small (e.g., 38 dots versus 40 dots). Critically, people differ in their precision at this task; some observers can reliably discriminate even very small ratios, but others require larger numerical differences between arrays in order to succeed. In adults and adolescents, individual differences in this simple, non-symbolic measure of ANS precision retrospectively correlate with scores on standardized math tests taken many years earlier, with various non-numerical factors controlled for (Halberda, Mazzocco, & Feigenson, 2008; Halberda et al., 2012; Libertus, Odic, & Halberda, 2012). In children, ANS precision correlates with concurrently measured symbolic math performance (Bonny & Lourenco, 2013; Guillaume, Nys, & Mussolin, 2013; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus, Feigenson, & Halberda, 2011; Lourenco, Bonny, Fernandez, & Rao, 2012; Odic et al., 2016). And in preschoolers, ANS precision measured at one time point (even as early as infancy) predicts future math performance (Chu & Geary, 2015; Libertus, Feigenson, & Halberda, 2013; Starr, Libertus, & Brannon, 2013; van Marle, Chu, Li, & Geary, 2014). Taken together, these findings suggest that the primitive, non-verbal sense of approximate number is linked with symbolic math from early in life.

However, not all examinations of the relationship between the ANS and symbolic math have found evidence of a correlation (Iuculano, Tang, Hall, & Butterworth, 2008; Price, Palmer, Battista, & Ansari, 2012; Sasanguie, DeSmedt, Defever, & Reynvoet, 2012). And for some of the cases in which a correlation was found, researchers have suggested that ancillary factors, rather than any direct link between the ANS and math performance, might be responsible. For example, Gilmore et al. (2013) suggested that individual differences in the ability to inhibit non-numerical dimensions when comparing arrays may underlie the relationship between the ANS and math performance (also Fuhs & McNeil, 2013, but for counter-evidence see Keller & Libertus, 2015). Others have argued that correlations between ANS precision and math are caused by individual differences in lower level visuospatial abilities (Tibber et al., 2013), although this does not explain the observed correlation between ANS precision (for sequences of tones) and math performance in congenitally blind individuals (Kanjlia, Feigenson, & Bedny, submitted). Still, although the relationship between the ANS and symbolic math abilities remains an active area of inquiry, meta-analyses suggest that, across the full body of studies of the ANS and math, there is a reliable relationship between the two (r = 0.24, 95% CI [0.14 0.26], Chen & Li, 2014; r = 0.24, 95% CI [0.20 0.28], Schneider et al., 2016).

Moreover, this link between ANS and symbolic math seems to go beyond the correlational. Emerging evidence finds that training observers in approximate number comparisons tasks not only improves their ANS precision, but also improves symbolic math performance (Hyde, Khanum, & Spelke, 2014; Park, Bermudez, Roberts, & Brannon, 2016; Park & Brannon, 2013; Wang, Odic, Halberda, & Feigenson, 2016). For example, we recently used a training task designed to either temporarily enhance or impair the ANS precision of 5-year old children. These children showed respective benefits or impairments in a subsequent symbolic math task, and no change in a subsequent verbal task.

But is the ANS linked with symbolic math ability in everyone? ANS representations and symbolic math ability have been found to correlate in children struggling with dyscalculia – a math-specific deficit (Landerl, Fusseneger, Mill, & Willburger, 2009; Mazzocco, Feigenson, & Halberda, 2011; Mejias, Grégoire, & Noël, 2012; Olsson, Ostergren, & Träff, 2016; Piazza et al., 2010; Skagerlund & Traff, 2016; but see also Iuculano et al., 2008). Many other studies have found a link in participants who exhibit symbolic math performance in the typical ability range (Anobile, Castaldi, Turi, Tinelli, & Burr, 2016; Bonny & Lourenco, 2013; Chu & Geary, 2015; Guillaume et al., 2013; Halberda et al., 2012; Halberda et al., 2008; Inglis et al., 2011; Libertus et al., 2012; Libertus et al., 2013; Lourenco et al., 2012; Odic et al., 2016; van Marle et al., 2014). But notably, studies of mathematically gifted individuals are missing from this picture.[2] How could such studies contribute to our understanding of the link between ANS precision and symbolic mathematics? Consider the widespread view that the ANS is a "primitive" system (i.e., a system that supports intuitive number thoughts in infants, and across animals and human cultures). This view may lead to theories of the causal role of the ANS in symbolic mathematics that focus on the early school years and on very basic math abilities (e.g., ordinal comparison); these theories might also predict that gifted students performing high-level mathematics will not show a link between their math performance and the ANS. For instance, one might expect that ANS representations will not play a major role in our understanding of and computing of binomial functions or cubed roots. In contrast, if the role of the ANS in symbolic math abilities extends beyond the early years and beyond 'primitive' operations, then we may observe a link between ANS performance and symbolic math abilities even in gifted students.

Here we asked whether individual differences in ANS precision are linked to math in high achieving individuals by studying adolescents enrolled in the Center for Talented Youth (CTY) program at the Johns Hopkins University. Each year, top 7th- and 8th-grade students (< 5% of the population) are invited by CTY to apply to their summer enrichment program. These students typically are initially selected because they performed above the 95th percentile in nationally normed tests or were recommended by teachers on the basis of exceptional academic records. The students are then screened using standardized college entrance exams such as the SAT or the ACT, which are usually administered when children are in 12th grade. Students qualify by achieving the required minimum scores on these tests – about the 50th

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[1] Even for animals trained to map numerical representations to symbols (Boysen & Berntson, 1989; Matsuzawa, 1985; Pepperberg, 1994), their performance suggests that they do not exhibit integer representations. Rather, these animals may have learned mappings between the ANS and digits or specific vocalizations.

[2] Although Mazzocco et al. (2011) found that adolescents identified as high achieving in math (> 95th percentile) exhibited better ANS precision than typically achieving children, this difference did not reach significance in their sample. However, their study focused on children at the lower end of the math performance distribution, and may have been underpowered to examine the relationship between ANS precision and math performance in high achievement.