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The contributions of numerical acuity and non-numerical stimulus features to the development of the number sense and symbolic math achievement

Ariel Starr[a,*], Nicholas K. DeWind[b], Elizabeth M. Brannon[b]

[a] Department of Psychology, University of California, Berkeley, United States
[b] Department of Psychology, University of Pennsylvania, United States

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ABSTRACT

Numerical acuity, frequently measured by a Weber fraction derived from nonsymbolic numerical comparison judgments, has been shown to be predictive of mathematical ability. However, recent findings suggest that stimulus controls in these tasks are often insufficiently implemented, and the proposal has been made that alternative visual features or inhibitory control capacities may actually explain this relation. Here, we use a novel mathematical algorithm to parse the relative influence of numerosity from other visual features in nonsymbolic numerical discrimination and to examine the strength of the relations between each of these variables, including inhibitory control, and mathematical ability. We examined these questions developmentally by testing 4-year-old children, 6-year-old children, and adults with a nonsymbolic numerical comparison task, a symbolic math assessment, and a test of inhibitory control. We found that the influence of non-numerical features decreased significantly over development but that numerosity was a primary determinate of decision making at all ages. In addition, numerical acuity was a stronger predictor of math achievement than either non-numerical bias or inhibitory control in children. These results suggest that the ability to selectively attend to number contributes to the maturation of the number sense and that numerical acuity, independent of inhibitory control, contributes to math achievement in early childhood.

1. Introduction

Beginning with Piaget, there has been great controversy regarding how children make decisions about number. In his classic number conservation studies, Piaget observed that although children correctly viewed two lines composed of the same number of objects as equally numerous when the spacing of the objects was equal, they erroneously judged one line as more numerous when the objects were spaced further apart (Piaget, 1952). As a result, Piaget concluded that children are unable to disentangle representations of number and space until they reach the concrete operations stage around seven years of age. More recent work, however, suggests that even infants can selectively attend to both number and size, particularly for large set sizes (see Cantrell & Smith, 2013; Mou & vanMarle, 2013 for reviews). At the same time, it is clear that numerical representations, even in adulthood, are influenced by non-numerical properties such as element size, field area (also referred to as convex hull), and density (Allik & Tuulmets, 1991; Dakin, Tibber, Greenwood, Kingdom, & Morgan, 2011; Defever, Reynvoet, & Gebuis, 2013; Gebuis & Gevers, 2011; Gebuis, Herfs, Kenemans, & van der Smagt, 2009; Gebuis & Reynvoet, 2011b, 2012a, 2012b; Ginsburg & Nicholls, 1988; Hurewitz, Gelman, & Schnitzer, 2006; Rousselle & Noël, 2008; Rousselle, Palmers, & Noël, 2004; Soltész, Szűcs, & Szűcs, 2010; Sophian & Chu, 2008; Szűcs, Nobes, Devine, Gabriel, & Gebuis, 2013; Tibber, Greenwood, & Dakin, 2012; Tokita & Ishiguchi, 2010, 2013). In general, numerical decisions are more accurate when these non-numerical features co-vary reliably with number compared to when they do not.

In most assessments of nonsymbolic numerical discrimination, the stimuli are constructed such that in half of the trials numerosity is congruent with area (i.e., the more numerous array contains larger individual elements and has a larger total surface area than the less numerous array). In the other half of trials, numerosity and area are incongruent such that the more numerous array has smaller elements and a smaller total surface area (e.g., DeWind &

* Corresponding author at: Department of Psychology, UC Berkeley, 134 Barker Hall, Berkeley, CA 94720, United States.
E-mail address: arielstarr@berkeley.edu (A. Starr).

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Brannon, 2012; Halberda & Feigenson, 2008; Inglis & Gilmore, 2013; Piazza et al., 2010). With this design, performance is typically better for congruent trials compared to incongruent trials. Furthermore, the effects of congruency are strongest for young children and attenuate with age, which suggests that younger children may be more biased by non-numerical cues than older children when making numerical comparison judgments (Defever, Sasanguie, Gebuis, & Reynvoet, 2011; Fuhs & Mcneil, 2013; Gilmore, Attridge, Clayton, et al., 2013; Rousselle & Noël, 2008; Rousselle et al., 2004; Soltész et al., 2010). For example, Soltész et al. (2010) tested 4- to 7-year-old children with a nonsymbolic magnitude comparison task in which the overall surface area or overall perimeter of the dot arrays was either congruent or incongruent with number. All children were more accurate when surface area and perimeter were congruent with number, but the difference in performance between congruent and incongruent trials decreased between 4 and 7 years of age. In addition, the precision with which numerical comparison judgments are made regardless of congruency condition follows a similar developmental trajectory, undergoing rapid development in infancy and early childhood and continuing to improve into adulthood (Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza et al., 2010).

Despite the wealth of research demonstrating that congruency effects decrease with age while numerical acuity improves, it is unclear whether these changes arise from a common mechanism or whether they represent differentiable developmental effects. Does overall improvement in numerical acuity over development reflect an increase in the precision of the internal numerical representations themselves or, alternatively, an increase in the ability to selectively attend to number and inhibit attention towards other stimulus features? A second open question concerns the relative influence of different types of non-numerical features on numerical discrimination. Does the influence of all non-numerical cues decrease in parallel, or do the trajectories vary by feature?

The development of selective attention to number may also have implications for the acquisition of symbolic math skills. Many studies have documented a relation between performance on numerical comparison tasks and symbolic math performance (DeWind & Brannon, 2012; Fazio, Bailey, Thompson, & Siegler, 2014; Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Halberda et al., 2012; Libertus, Feigenson, & Halberda, 2011; Libertus, Odic, & Halberda, 2012; Mazzocco, Feigenson, & Halberda, 2011; Mussolin, Nys, Leybaert, & Content, 2012; Starr, Libertus, & Brannon, 2013). However, a growing number of studies have observed no relation (Holloway & Ansari, 2009; Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013; Sasanguie, Defever, Maertens, & Reynvoet, 2013) or a relation that exists only in young children or children with low math ability (Bonny & Lourenco, 2012; Inglis, Attridge, Batchelor, & Gilmore, 2011; Sasanguie, De Smedt, Defever, & Reynvoet, 2011). Three recent meta-analyses on this topic concluded that there is a significant yet small relation between symbolic math and non-symbolic numerical acuity that is strongest in young children (Chen & Li, 2013; Fazio et al., 2014; Schneider et al., 2016), lending credence to the hypothesis that approximate number representations may be foundational for the acquisition of symbolic math (Dehaene, 1997; Gallistel & Gelman, 1992). According to this view, having a more precise internal representation of number is advantageous for learning numerical symbols and symbolic arithmetic operations.

An alternative theory, however, is that the apparent relation between numerical acuity and math ability actually reflects the well-documented relation between inhibitory control and math ability (e.g., Bull & Lee, 2014; Bull & Scerif, 2001; St Clair-Thompson & Gathercole, 2006). One proposal is that the need to attend to number and ignore irrelevant non-numerical features in nonsymbolic numerical comparison tasks taps inhibitory control as much as, or to an ever greater extent, than numerical acuity (Fuhs & Mcneil, 2013; Gilmore, Attridge, Clayton, et al., 2013). According to this view, children (and adults) with stronger inhibitory control perform more accurately on nonsymbolic numerical comparison tasks because they are better able to inhibit attention towards non-numerical stimulus features and focus their attention on number. Likewise, this stronger inhibitory control also contributes to higher math performance. Therefore, if better performance on nonsymbolic numerical comparison tasks is actually reflecting inhibitory control rather than the precision of internal numerical representations, numerical acuity in and of itself may not directly contribute to math achievement.

The present study was designed to investigate how children's attention to numerical and non-numerical stimulus features shifts over development and the degree to which attention to these features is related to symbolic math performance. We tested 4- and 6-year-old children, as well as adults, on a nonsymbolic numerical comparison task, a symbolic math assessment, and an inhibitory control task. The nonsymbolic numerical comparison task and analysis method were adapted from DeWind, Adams, Platt, and Brannon (2015). This method is unique in its treatment of non-numerical stimulus features (e.g. field area, total surface area, and individual element size). Instead of defining stimulus subsets that control for a particular feature by holding it constant (Halberda & Feigenson, 2008) or trying to minimize the correlation between number and all non-numeric features overall (Gebuis & Reynvoet, 2011a), non-numerical stimulus features are intentionally varied, such that their effect on numerical discrimination can be explicitly modeled. Critically, this enables us to go beyond the dichotomy of congruent and incongruent trials to quantitatively estimate how different non-numerical features influence numerical decision-making. The model, fit to each participant's data, returns coefficients that represent the effect of number and non-numerical size and spacing features on ostensibly numerical discriminations. By comparing the coefficients across different age groups, we can examine how the influence of different non-numerical features changes over development. This framework also allows an explicit test of the hypothesis that the ability to inhibit attention towards non-numerical features drives the relation between performance on nonsymbolic numerical comparison tasks and symbolic math skill through an investigation of the relation between the non-numerical feature coefficients, symbolic math performance, and inhibitory control.

2. Method

2.1. Participants

Thirty-nine 4-year-old children (mean age 4.64 years, range: 4.45–4.96 years, 17 females), 45 6-year-old children (mean age 6.63 years, range: 6.29–6.96 years, 19 females), and 30 adults (mean age 21.9 years, range 18.5–54.4 years, 20 females) participated in the study. An additional 11 4-year-olds were excluded due to failure to complete all tasks (n = 10), or inability to speak English (n = 1), and one 6-year-old was excluded due to failure to complete all tasks. All participants or their parents consented to a protocol approved by the local IRB. Children were given a small gift and parents were compensated monetarily at each visit. Adults were compensated either monetarily or with course credit.