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Do analog number representations underlie the meanings of young children's verbal numerals?

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ABSTRACT

Children learn to count, and even learn the cardinal meanings of the first three or four verbal numerals ("one" through "three" or "four"), *before* they master the numerical significance of counting. If so, it follows that the cardinal meanings of those first few numerals cannot be derived, initially, from their place in the count list and the counting routine. What non-verbal representations, then, support the cardinal meanings of verbal numerals before children have mastered how counting does so? Four experiments addressed the commonly adopted assumption that in the earliest period of learning the meanings of number words, children map verbal numerals to regions of the analog number system (ANS), a system of representation with numerical content that is widely attested in animals and in human infants. Experiment 1 confirmed that children who know what "three" means, but who do not yet know what "four" means, and do not yet know how counting represents number, can be easily taught the meaning of "four," if they are trained to indicate sets of four when they are paired with a series of sets that contrast numerically with four. If children learn "four" by mapping the word to an ANS representation of sets of four, and if such ANS value-to-word mappings underlie the meanings of other known numerals early in development, then analogous teaching should enable young children to establish a ANS value-to-word mapping for between "ten" and sets of 10 as specified by the ANS. Furthermore, the ease of learning should be a function of the ratio of the number of individuals in the comparison set to 10. Three further experiments tested these hypotheses by attempting to teach young Cardinal Principle-knowers the meaning of the word "ten," under the same training conditions "three-"knowers are easily taught the meaning of "four". The children learned which *picture* in each training pair had "ten." However, test trials with novel animals and spatial configurations showed that they had failed to learn what *set sizes* should be labeled "ten", even when, after training, they were asked to indicate a set of 10 vs. a set of 20 or 30 (well within the ratio sensitivity of the ANS even early in infancy). Furthermore, there was no effect of ratio on success during test trials. These data provide new evidence that ANS value-to-word mappings do not underlie the meanings of number words early in development. We discuss what other non-verbal representations might do so, and discuss other ways the ANS may support learning how counting represents number.

1. Introduction

Mathematics does not come for free by virtue of being born a human being. Historically, the cultural construction of mathematics began with arithmetic (Dantzig, 1967; Ifrah, 1985). As the foundational concepts in arithmetic are the positive integers, a good place to start in understanding the *ontogenesis* of mathematics is to account for the ontogenetic origin of representations of the positive integers. In the first systematic attempt at such an account, Piaget (1952) argued that integer representations must await the logical developments of concrete operational thought. He offered non-conservation of number by preoperational children as evidence that concepts of integers do not become available until age 5 or 6.

In the first major re-evaluation of Piaget's position, Gelman and Gallistel (1978) countered that verbal counting, when deployed in accordance with the counting principles of stable order, 1–1 correspondence and the cardinality principle, constitutes a representation of at least a finite subset of the positive integers. Gelman

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and Gallistel provided evidence for mastery of these three counting principles by young 2-year-olds, and proposed that learning to count is supported by an innate "numeron" list, used in accord with the counting principles to represent cardinal number. Learning to count in a natural language, according to this hypothesis, requires only that the child identify what ordered list of words should be mapped to the innate numeron list. Thus, Gelman and Gallistel, in contrast to Piaget, argued that the positive integers are innate.

Subsequent work has undermined the empirical support for an innate count list. Two year olds indeed know the count routine, and deploy it in stable order and in 1–1 correspondence to the individuals counted, but much evidence suggests they do not know that the last word reached in a count represents the cardinal value of the set (the cardinal principle) until months or even years later (Fuson, 1988; Le Corre, Van de Walle, Brannon, & Carey, 2006; Mix, Huttenlocher, & Levine, 2002; Sarnecka & Lee, 2009; Siegler, 1991; Wynn, 1990; Wynn, 1992; see Carey, 2009 for review). Rather, they assign numerical meaning to the verbal numerals in a piecemeal way, first learning what "one" means (i.e., become "one"-knowers), then some 6 months later become "two"-knowers, then "three"-knowers, and then "four"-knowers. Children who know only the meanings of some of the numerals between "one" and "four" are designated "subset-knowers," for they know the cardinal meanings of only a subset of the numerals they can recite. Middle-class, English learning, children become cardinal principle-knowers (CP-knowers) around age 3 ½ to 4 ½, and can then use the count routine to assign a cardinal value to any words in their known and practiced count list (Gunderson, Spaepen, & Levine 2015; Le Corre & Carey, 2007; Sarnecka & Lee, 2009).

If we accept that children in the subset-knower period do not know the significance of counting, it follows the cardinal content of the words "one" through "four" in the subset-knower cannot be provided by their role in a counting procedure constrained by the counting principles (e.g., the numeral "four" cannot receive its meaning by virtue of being the fourth word in the count list). This conclusion raises an important question: if the meaning of the first verbal numerals is not provided by their role in counting, how *do* they get their numerical content?

One likely source of number word meanings is antecedently available *non-verbal* representations of number. It is very difficult to see how meanings for number words might be constructed entirely from representations with no numerical content. Indeed, non-human animals, human infants, children, and adults share two quite different evolutionarily ancient systems of non-verbal representations with numerical content: (1) the analog, or approximate, number system (ANS); and (2) parallel individuation (PI), a structure in which working memory models of small sets of individuals are constructed with one symbol in the model for each individual in the set. These are the only two systems of nonverbal representations with numerical content for which there is evidence in non-human animals and very young human infants (see Carey, 2009; Feigenson, Dehaene, & Spelke, 2004, for review).

The ANS consists of analog representations that are a linear or logarithmic function of the cardinal values of the set represented. These representations express cardinal values only approximately. One signature of the ANS is that magnitudes are discriminated one from another on the basis of their ratios; thus, discriminability accords with Weber's law and exhibits scalar variability (the standard deviation of multiple estimates of a the number of items in a set is a linear function of that set's cardinal value). ANS representations support many different arithmetical computations, including numerical comparison, addition, subtraction, multiplication, and division (see Carey, 2009; Dehaene, 2011; Gallistel, 1990, for review).

PI, a second preverbal system with numerical content, consists of working memory representations of small sets of individuals. The symbols in this system represent individuals (e.g., a set of three crackers is represented CRACKER, CRACKER, CRACKER, probably iconically for each cracker). Unlike the ANS, the PI working memory system is not a dedicated number representation system, *nor are there any symbols* that represent cardinal values in these models; there are only symbols for individuals, held in working memory. The numerical content in PI is implicit, carried by the computations that ensure that the symbols in a working memory model stand in one-to-one correspondence with the individuals in the sets modeled, and the computations that allow models to be compared on the basis of one-to-one correspondence to determine numerical equivalence. There is a strict upper limit to capacity of working memory, a function of the number of encoded individuals, the complexity of the representations of individuals, and the complexity of the computations to which the models serve as input (Brady & Alvarez, 2015; Xu & Chun, 2009; Zosh & Feigenson, 2009). For twelve-month-olds, this limit on working memory representations of single sets is three distinct, perceptually simple individuals (Feigenson & Carey, 2003; Ross-Sheehy, Oakes, & Luck, 2003); with development this capacity expands a bit, reaching a limit of four or five in older preschoolers (Starkey & Cooper, 1995).

Although researchers for the most part have abandoned the hypothesis of an innate numeron list and counting routine, almost all agree with Gelman and Gallistel's crucial insight that the count list, deployed in accord with the counting principles, constitutes a representation of at least a subset of the positive integers. Furthermore, neither preverbal representational system, on its own, is capable of expressing integers: the ANS because it only approximates cardinal values and does not naturally implement the successor function, and PI because it contains no symbols for cardinal values and has a capacity limit on the size of sets it can represent. Thus, much work in the field concerns the process through which children learn the cardinal principle, as the counting principles ensure that verbal numerals *do* represent integers. All theories posit innate numerical resources *in addition to PI and the ANS*; examples include an innate successor function (Leslie, Gelman, & Gallistel, 2007); an innate tally system based on the iteration of 1 (Leslie, Gelman, & Gallistel, 2008); and quantification in natural language morpho-syntax and semantics (Almoammer et al., 2013; Barner, Libenson, Cheung, & Takasaki, 2009; Bloom & Wynn, 1997; Le Corre, Li, Huang, Jia, & Carey, 2016; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007). All suggest that some process of combining or aligning antecedently independent representational systems is likely involved (e.g., Carey, 2009; Leslie et al., 2007, 2008; Spelke, 2003). In order to evaluate these different proposals, we need to know what *non-verbal* representations underlie children's meanings of the first number words, for those meanings will play a central role in the construction of explicit representations of positive integers.

Here we assume that the two well attested systems of representation are the only *preverbal* representations with numerical content available to underlie the meanings of the words "one" through "four" in the subset-knower stage. This is because there is no *evidence* for an innate successor function or an innate tally system based on the iteration of 1. Our question is whether one system, or both, do so, and how. Several writers presuppose (Bugden & Ansari, 2011; Odic, Le Corre, & Halberda, 2015; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013) and/or explicitly argue (Dehaene, 2009; Gallistel & Gelman, 2000; Piazza, 2010; Starr, Libertus, & Brannon, 2013; Verguts & Fias, 2004; Wagner & Johnson, 2011) that the ANS provides such meanings, and it does so though the creation of mappings of each number word "one" through "four" with an ANS region (as ANS values can be specified