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Wormholes in virtual space: From cognitive maps to cognitive graphs

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ABSTRACT

Humans and other animals build up spatial knowledge of the environment on the basis of visual information and path integration. We compare three hypotheses about the geometry of this knowledge of navigation space: (a) 'cognitive map' with metric Euclidean structure and a consistent coordinate system, (b) 'topological graph' or network of paths between places, and (c) 'labelled graph' incorporating local metric information about path lengths and junction angles. In two experiments, participants walked in a non-Euclidean environment, a virtual hedge maze containing two 'wormholes' that visually rotated and teleported them between locations. During training, they learned the metric locations of eight target objects from a 'home' location, which were visible individually. During testing, shorter wormhole routes to a target were preferred, and novel shortcuts were directional, contrary to the topological hypothesis. Shortcuts were strongly biased by the wormholes, with mean constant errors of 37° and 41° (45° expected), revealing violations of the metric postulates in spatial knowledge. In addition, shortcuts to targets near wormholes shifted relative to flanking targets, revealing 'rips' (86% of cases), 'folds' (91%), and ordinal reversals (66%) in spatial knowledge. Moreover, participants were completely unaware of these geometric inconsistencies, reflecting a surprising insensitivity to Euclidean structure. The probability of the shortcut data under the Euclidean map model and labelled graph model indicated decisive support for the latter ($BF_{GM} > 100$). We conclude that knowledge of navigation space is best characterized by a labelled graph, in which local metric information is approximate, geometrically inconsistent, and not embedded in a common coordinate system. This class of 'cognitive graph' models supports route finding, novel detours, and rough shortcuts, and has the potential to unify a range of data on spatial navigation.

1. Introduction

As humans and other animals explore their environments, they build up spatial knowledge based on visual, idiothetic, and other sensory information. The underlying geometry of the resulting knowledge might take a number of forms (Tobler, 1976; Trullier, Wiener, Berthoz, & Meyer, 1997; Tversky, 1993). At one end of the spectrum (Fig. 1) lies a Euclidean cognitive map, which preserves metric information about the locations of known places in a common coordinate system (Gallistel, 1990; O'Keefe & Nadel, 1978; Tolman, 1948). At the other end lies weak topological structure, such as a graph that only preserves a network of paths connecting known places (Byrne, 1979; Kuipers, Tecuci, & Stankiewicz, 2003; Werner, Krieg-Brückner, & Herrmann, 2000). Various combinations of metric and topological knowledge have also been proposed, capitalizing on the advantages of each (Chown, Kaplan, & Kortenkamp, 1995; Kuipers, 2000; Mallot & Basten, 2009; Meilinger, 2008; Poucet, 1993). After decades of research on this issue, researchers still hold opposing views and the question remains unresolved. In this article we report two experiments on navigation in a non-Euclidean environment that challenge both extremes. We argue that the evidence is best accounted for by a *labelled graph* that incorporates local metric information but has no globally consistent coordinate system.

1.1. Euclidean maps

Euclidean knowledge (Fig. 1A) would be advantageous because it supports flexible navigation, including novel as-the-crow-flies shortcuts and the integration of separately learned routes. An influential theory holds that a metric Euclidean map is constructed on the basis of path integration (Gallistel & Cramer, 1996; McNaughton, Battaglia, Jensen, Moser, & Moser, 2005; O'Keefe & Nadel, 1978). Specifically, as an animal explores the environment, the path integrator registers idiothetic (i.e. proprioceptive, motor,

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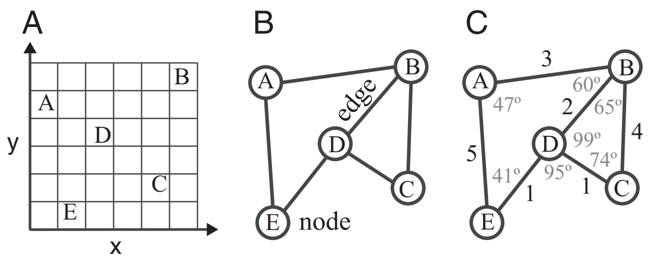

Fig. 1. Models of spatial knowledge. (A) Euclidean map: places are assigned locations in a common coordinate system. (B) Topological graph: nodes correspond to places and edges to paths between them. (C) Labelled graph: edge weights denote approximate path lengths and node labels denote approximate junction angles.

vestibular) information about the angles turned and distances travelled from a home location, and assigns salient places to coordinates in this inertial coordinate system. Grid, place, and head-direction cells in the hippocampal formation have been interpreted as a system for encoding metric maps from path integration (Derdikman & Moser, 2010; McNaughton et al., 2005). Indeed, mammals and insects have been observed to take shortcuts between known locations (Chapuis, Durup, & Thinus-Blanc, 1987; Cheeseman et al., 2014; Gould, 1986; Menzel et al., 2006), and humans are able to estimate the directions and distances between familiar places (Chrastil & Warren, 2013; Holmes & Sholl, 2005; Ishikawa & Montello, 2006; Schinazi, Nardi, Newcombe, Shipley, & Epstein, 2013; Waller & Greenauer, 2007; Weisberg, Schinazi, Newcombe, Shipley, & Epstein, 2014), consistent with a Euclidean cognitive map.

On closer examination, however, the evidence appears inconclusive. Apparently novel shortcuts may be more simply explained by knowledge of familiar routes, landmarks, or views (Benhamou, 1996; Bennett, 1996; Cheung et al., 2014; Dyer, 1991; Foo, Warren, Duchon, & Tarr, 2005). Directional estimates in humans are highly unreliable, with absolute angular errors of 20–100° and angular standard deviations on the order of 30° (Chrastil & Warren, 2013; Foo et al., 2005; Ishikawa & Montello, 2006; Meilinger, Riecke, & Bülthoff, 2014; Schinazi et al., 2013; Waller & Greenauer, 2007; Weisberg et al., 2014), while junctions tend to be orthogonalized to 90° (Byrne, 1979). Distance estimates are biased by the number of intervening junctions, turns, and boundaries, and are asymmetric between more and less salient places (Burroughs & Sadalla, 1979; Byrne, 1979; Cadwallader, 1979; Kosslyn, Pick, & Fariello, 1974; McNamara & Diwadkar, 1997; Sadalla & Magel, 1980; Sadalla & Staplin, 1980; Tversky, 1992). People often fail to integrate learned routes, and cross-route estimates are generally poor (Golledge, Ruggles, Pellegrino, & Gale, 1993; Ishikawa & Montello, 2006; Moeser, 1988; Schinazi et al., 2013; Weisberg et al., 2014).

The *sine qua non* of a Euclidean map is a distance metric that satisfies the metric postulates. A metric space is defined by a distance metric that must satisfy the postulates of *positivity* (AA = 0, AB > 0), *symmetry* (AB = BA), segmental *additivity* (AB + BC = AC), and the *triangle inequality* (AB + BC ≥ AC), where pairs of letters denote distances between pairs of points (Beals, Krantz, & Tversky, 1968). Yet the human distance and direction estimates reviewed above imply violations of the postulates of symmetry, additivity, and the triangle inequality.

Given its supposed ubiquity, there is thus a surprising lack of convincing evidence for a metric Euclidean map. However, it is difficult to reject the hypothesis because the expected level of performance is not well-specified. The view thus remains influential and has many prominent advocates (Byrne, Becker, & Burgess, 2007; Cheeseman et al., 2014; McNaughton et al., 2005; Nadel, 2013).

1.2. Topological graphs

At the other extreme lies a topological graph (Fig. 1B), a network of connections in which nodes correspond to familiar places and edges to known paths between them.[1] Graph knowledge would be advantageous because available routes and detours are explicitly specified in one compact structure, and do not need to be derived from a coordinate map via additional operations. Nodes may have associated place information such as views, landmarks, or surface layout (local geometry), enabling self-localization and orientation (Epstein & Vass, 2014). It has been suggested that place cell activity might reflect a hippocampal graph (Dabaghian, Mémoli, Frank, & Carlsson, 2012; Muller, Stead, & Pach, 1996). Place fields are anchored to environmental features and their metric locations shift with transformations of the layout (Dabaghian, Brandt, & Frank, 2014; Muller & Kubie, 1987; O'Keefe & Burgess, 1996).

Graph knowledge is richer than route knowledge, but weaker than 'survey' or map knowledge. Whereas routes are typically characterized as chains of place-action associations (Trullier et al., 1997), a graph can express multiple routes between two places and multiple paths intersecting at one place. Thus, graph knowledge provides the basis for finding novel routes and detours (Chrastil & Warren, 2014). On the other hand, a purely topological graph is insufficient to explain behavior such as taking shorter routes or novel shortcuts, implying that topological knowledge may be augmented by metric information.

These considerations have led to hybrid theories in which topological and map knowledge are represented in parallel or hierarchical systems (Byrne et al., 2007; Chown et al., 1995; Kuipers, 2000; Thrun, 1998; Trullier et al., 1997). However, such models are less parsimonious and are compromised by the lack of evidence for metric maps.

1.3. Labelled graphs

We suggest that spatial knowledge is more appropriately characterized by a labelled graph, a single structure that incorporates local metric information (Fig. 1C). Specifically, path lengths are denoted by edge weights and the angles between paths at junctions are denoted by node labels. This local metric information is typically noisy, biased, and geometrically inconsistent. Yet such a *cognitive graph* is sufficient to find generally shorter routes and detours, and to generate approximate shortcuts, although their accuracy would depend on the level of error in the graph.

What distinguishes a labelled graph from a metric map is that the local information is not embedded into a common coordinate system, a 'global metric embedding' in which places are assigned coordinates in a globally consistent map. Apparently Euclidean behavior does not necessarily imply a metric map, for it could be underwritten by a labelled graph together with adaptive navigation strategies. For example, approximate shortcuts could be generated by vector addition along a path through the graph,[2] which may be sufficient to bring the navigator within sight of local beacons, yielding successful shortcuts between familiar places (Foo et al., 2005).

A labelled graph is reminiscent of previous theories that combine metric and topological information in a single graph structure (Mallot & Basten, 2009; Meilinger, 2008; Poucet, 1993). These models are based on the concept of local reference frames (or coordinate systems) that are linked by vectors specifying the metric distance and direction between them, and are designed to solve

[1] A graph can also describe connections between other entities, such as views (Hübner & Mallot, 2007) or neighborhoods (Wiener & Mallot, 2003).

[2] Formally, vector addition can be performed in a coordinate-free space by iterative application of the parallelogram law and cosine and sine rules.