Cognition 160 (2017) 82–90

Contents lists available at ScienceDirect

Cognition

journal homepage: www.elsevier.com/locate/COGNIT

Original Articles

A dissociation between small and large numbers in young children's ability to "solve for x" in non-symbolic math problems

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ARTICLE INFO

Article history:
Received 12 February 2016
Revised 2 December 2016
Accepted 20 December 2016
Available online 6 January 2017

Keywords:
Approximate number system
Objects
Number
Numerical cognition
Mathematical cognition
Cognitive development

ABSTRACT

Solving for an unknown addend in problems like 5 + x = 17 is challenging for children. Yet, previous work (Kibbe & Feigenson, 2015) found that even before formal math education, young children, aged 4- to 6-years, succeeded when problems were presented using non-symbolic collections of objects rather than symbolic digits. This reveals that the Approximate Number System (ANS) can support pre-algebraic intuitions. Here, we asked whether children also could intuitively "solve for x" when problems contained arrays of four or fewer objects that encouraged representations of individual objects instead of ANS representations. In Experiment 1, we first confirmed that children could solve for an unknown addend with larger quantities, using the ANS. Next, in Experiment 2a, we presented addend-unknown problems containing arrays of four or fewer objects (e.g., 1 + x = 3). This time, despite the identical task conditions, children were unable to solve for the unknown addend. In Experiment 2b, we replicated this failure with a new sample of children. Finally, in Experiment 3, we confirmed that children's failures in Experiments 2a and b were not due to lack of motivation to compute with small arrays, or to the discriminability of the quantities used: children succeeded at solving for an unknown *sum* with arrays containing four or fewer objects. Together, these results suggest that children's ability to intuitively solve for an unknown addend may be limited to problems that can be represented using the ANS.

1. Introduction

Children are introduced to formal mathematics starting in early elementary school, but the process of acquiring formal mathematical skills is protracted. One reason formal math is thought to be particularly challenging for children is that it requires young learners to mentally manipulate symbols according to a set of learned rules (Kieran, 1992; Nathan, 2012; Susac, Bubic, Vrbanc, & Planinic, 2014; Van Amerom, 2003). For example, a child encountering the problem $2 + 3 = x$ must understand the meanings of the symbols (digits and operators) and the algorithm for combining the two digits as specified by the operator symbol. Misunderstanding of or difficulty processing the meanings of mathematical symbols predicts poorer mathematical performance in children (Byrd, McNeil, Chesney, & Matthews, 2015; Desoete, Ceulemans, De Weerdt, & Pieters, 2012). And as mathematics becomes more complex over successive years of instruction, requiring the manipulation of variables as well as digits and operators, learners continue to struggle even into the college years (Koedinger, Alibali, & Nathan, 2008).

Although learning to manipulate the symbols used in formal mathematics is challenging, infants, children, adults, and non-human animals have fundamental mathematical intuitions that do not depend on external symbols. These populations all share an Approximate Number System (ANS) that allows them to estimate the number of items in visual and auditory arrays without language, education, or symbolic notation (e.g., Dehaene, 1997; Feigenson, Dehaene, & Spelke, 2004; Libertus & Brannon, 2009). Unlike the exact number representations involved in most symbolic math, the number representations generated by the ANS are noisy and imprecise—this remains true throughout the lifespan, even after children have learned to represent exact number symbolically (Carey, 2009; Halberda & Feigenson, 2008a; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Xu & Spelke, 2000).

The nature of the relationship between the ANS and acquired school mathematical abilities remains the topic of much debate. However, evidence suggests that the ANS plays a role in school math achievement, despite most of school mathematics requiring the kinds of precise representations that the ANS lacks. First, individual differences in the precision of the ANS correlate with and

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http://dx.doi.org/10.1016/j.cognition.2016.12.006
0010-0277/

predict symbolic math performance (e.g., Chen & Li, 2014; DeWind & Brannon, 2012; Feigenson, Libertus, & Halberda, 2013; Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Halberda et al., 2012; Libertus, Feigenson, & Halberda, 2011; Starr, Libertus, & Brannon, 2013, but see also, e.g., Holloway & Ansari, 2009; Iuculano, Tang, Hall, & Butterworth, 2008; Soltesz, Szucs, & Szucs, 2010). Second, training of numerical approximation abilities has been found to improve symbolic math performance in adults and children (Hyde, Khanum, & Spelke, 2014; Park, Bermudez, Roberts, & Brannon, 2016; Park & Brannon, 2013, 2014; Wang, Odic, Halberda, & Feigenson, 2016).

One way in which the ANS might be useful during the process of initially learning formal mathematics is by providing basic intuitions about numerical computations. Indeed, despite their noisiness, ANS representations can support many of the computations that are later encountered in formal mathematics, including ordering (Lipton & Spelke, 2005), addition and subtraction (Barth et al., 2006; Booth & Siegler, 2008; Gilmore, McCarthy, & Spelke, 2007; McCrink & Wynn, 2004), multiplication (McCrink & Spelke, 2010), and division (McCrink & Spelke, 2016). Critically, recent research suggests that presenting problems non-symbolically, using arrays that encourage the use of ANS representations, can help children solve at least some of the more complex computations that are used in formal schooling – even problems that many children struggle with into adolescence. In this previous work (Kibbe & Feigenson, 2015), we found that, not surprisingly, 4- to 6-year-old children failed to solve symbolically presented pre-algebraic problems (i.e., problems that required solving for an unknown addend, like "6 + x = 18," presented using digits). Yet these children spontaneously "solved for x" when the very same problems were presented non-symbolically using collections of objects. In these studies, children were introduced to a "magic cup" that always transformed object collections by a constant quantity. Then they saw a starting quantity (e.g., six objects), watched as the magic cup was applied to that quantity, and finally saw a new quantity (e.g., 18 objects) revealed. After seeing events like this, children were able to correctly infer the approximate quantity in the magic cup—in this sense, they solved for the value of the unknown addend *x*.

These results suggest that presenting problems non-symbolically, with collections of objects instead of written or spoken number symbols, can sometimes help children perform specific mathematical computations earlier than they otherwise could. Harnessing ANS representations appears to allow children to form "gut-sense" estimates of the quantities involved, even when the quantities' values had to be inferred. However, ANS representations are limited in some important ways. Whereas symbolically mediated exact number representations allow children to form very precise representations of x in a "solve for x" task, ANS representations inherently provide only noisy estimates. These estimates were sufficiently precise to allow children to succeed in our "magic cup" task – for example, after seeing six buttons transformed by the magic cup to yield 18 buttons, children correctly identified the cup as containing 12 buttons rather than 4 or 24. Distinguishing between 4 and 12, or 12 and 24, can be accomplished even from noisy estimates. But ANS representations, because of their inherent imprecision, should not support discrimination of the target from numerically nearer distractors (e.g., 12 versus 13 buttons).

ANS representations may pose yet a further limitation on children's ability to solve for unknown variables. Much evidence suggests that whereas ANS representations are readily deployed in response to large quantities (usually quantities greater than three), they often fail to be deployed in response to smaller quantities. Instead, young children presented with one, two, or three items often appear to represent these arrays in terms of individual objects (Object A, Object B, Object C) rather than as a single entity with an approximate (or exact) cardinality (Coubart, Izard, Spelke, Marie, & Streri, 2014; Feigenson & Carey, 2003, 2005; Feigenson, Carey, & Hauser, 2002; Feigenson, Carey, & Spelke, 2002; Hyde & Spelke, 2011; vanMarle, 2013; Xu, 2003). Although under some circumstances infants can be induced to represent arrays of one, two, or three objects using approximate number representations (e.g., Cordes & Brannon, 2009), small and large arrays often appear to trigger the deployment of two separate representational systems. An open question, then, is whether children can solve for the value of an unknown variable using individual object representations rather than approximate number (ANS) representations. If children can "solve for x" with small quantities as well as large ones, this would suggest that pre-algebraic computations can be performed over multiple types of quantity-relevant representations, as long as external symbols (digits or words) are not required.

Here we tested this possibility by contrasting children's ability to non-symbolically "solve for x" with large versus small numbers of objects. We tested children of the same age as in our previous study, using the same non-symbolic "magic cup" task (Kibbe & Feigenson, 2015). First, in Experiment 1, we sought to replicate children's success at solving for the value of an unknown addend when the quantities involved large numerosities only. Next, in Experiment 2a, we asked whether children also could solve for x with small quantities of four or fewer – quantities that have been found by previous work to activate the system for representing individual objects rather than approximate cardinalities. To preview, we found that children succeeded in Experiment 1, but failed in Experiment 2a. In Experiment 2b, we replicated children's failure to solve for x with small quantities with a separate sample of children. Finally, in Experiment 3, we asked whether children's failures in Experiments 2a and 2b were due to representing transformations over small quantities, versus performing pre-algebraic computations. We found that when children were asked to solve for the value of an unknown sum, rather an unknown addend, they succeeded. We close by discussing the implications of these results for our understanding of children's early numerical abilities.

2. Experiment 1: Unknown addend, large quantities

The purpose of Experiment 1 was to replicate the finding that 4- to 6-year old children can solve for x when presented with non-symbolic arrays containing large numbers of objects. Children were introduced to a magic cup and were told that this cup always added the same number of objects to an existing collection. Children then saw the magic cup demonstrated on three different starting quantities (i.e., the cup added *x* to three different starting arrays). Finally, children were asked to choose which of two non-symbolic quantities the magic cup contained – i.e., they were asked to solve for *x*.

2.1. Participants

Twenty-four children (mean age: 5 years, 6 months; range: 4 years 1 month – 6 years 11 months; 10 girls) participated in the children's wing of a local science museum. Children received a sticker for their participation.

2.2. Methods

2.2.1. Materials

Materials consisted of a small stuffed alligator toy and a 10-oz white paper cup. The cup could transform the quantity of three different types of arrays: buttons, pennies, and small toy shoes. A second 10-oz white paper cup and a set of pink and purple pom-poms