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Original Articles

Preschool children use space, rather than counting, to infer the numerical magnitude of digits: Evidence for a *spatial mapping principle*

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A B S T R A C T

A milestone in numerical development is the acquisition of counting principles which allow children to exactly determine the numerosity of a given set. Moreover, a canonical left-to-right spatial layout for representing numbers also emerges during preschool. These foundational aspects of numerical competence have been extensively studied, but there is sparse knowledge about the interplay between the acquisition of the cardinality principle and spatial mapping of numbers in early numerical development. The present study investigated how these skills concurrently develop before formal schooling. Preschool children were classified according to their performance in Give-a-Number and Number-to-position tasks. Experiment 1 revealed three qualitatively different groups: (i) children who did not master the cardinality principle and lacked any consistent spatial mapping for digits, (ii) children who mastered the cardinality principle and yet failed in spatial mapping, and (iii) children who mastered the cardinality principle and displayed consistent spatial mapping. This suggests that mastery of the cardinality principle does not entail the emergence of spatial mapping. Experiment 2 confirmed the presence of these three developmental stages and investigated their relation with a digit comparison task. Crucially, only children who displayed a consistent spatial mapping of numbers showed the ability to compare digits by numerical magnitude. A congruent (i.e., numerically ordered) positioning of numbers onto a visual line as well as the concept that moving rightwards (in Western cultures) conveys an increase in numerical magnitude mark the mastery of a *spatial mapping principle*. Children seem to rely on this spatial organization to achieve a full understanding of the magnitude relations between digits.

1. Introduction

The development of a symbolic system to represent numerical quantities can be considered one of the most powerful cultural inventions of humans. The shift from an iconic to a symbolic notation has allowed individuals to efficiently denote and manipulate numerical quantities using a variety of transformations (Wiese, 2003), from simple arithmetical operations (i.e., addition and subtraction) to advanced mathematical procedures. From two years of age, young children begin to construct a stable connection between exact numerosities and symbolic representations of numerical quantities. Initially, this mapping is established between number-words and the exact numerosties through the acquisition of the counting principles (Gelman & Gallistel, 1978). At least three counting principles must be respected to correctly count the items of a given set: (a) the *stable-order* principle states that the list of number-words must be recited in the correct (received) order (i.e., 1, 2, 3, 4...); (b) the *one-to-one* correspondence principle claims that each object in the set must be associated with only one number-word in the counting list; and (c) the *cardinality* principle states that the last recited number-word identifies the number of elements in the set. The correct implementation of these principles allows children to determine the exact numerosity of a given set, thereby creating a meaningful connection between number words and the corresponding objective numerosities. The Give-a-Number (henceforth GaN; Wynn, 1990) is a well-established task to assess the acquisition of cardinality principle in young children. In this task, the experimenter repeatedly asks the child to give a

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specific number of items drawn from a larger set of objects (e.g., give 3 cookie toys from a basket containing 10 or more cookies). Children's performance shows a stable developmental pattern which follows the acquisition of the cardinal meaning of number-words (c.f. Knower-level theory, Carey, 2001; Sarnecka & Carey, 2008). At first, children grab a handful of items irrespective of the requested number: These children lack any numerical meaning of number-words and they are referred to as pre-number knowers (PN-knowers). Subsequently, children learn the cardinal meaning of the number-word "one" (i.e., one-knowers) and they correctly provide one item when requested. Interestingly, when requested for a larger numerosity, one-knowers unlikely provide one item because they know the cardinal meaning of the number-word "one". Later, children can correctly give two objects, but they are still unsuccessful with larger numerosities (i.e., two-knowers). With practice, children learn the cardinal meaning of number-words up to four, thereby moving from a PN-knower level to a four-knower level. These children are also denoted as subset-knowers because their cardinal knowledge is limited to a subset from 1 to 4 (Condry & Spelke, 2008; Le Corre & Carey, 2007; Le Corre, Van de Walle, Brannon, & Carey, 2006; Sarnecka & Gelman, 2004; Wynn, 1990, 1992). Children become cardinal-principle knowers (CP-knowers) when they understand that the next number-word in the count list corresponds to one additional element in the set (i.e., $n + 1$). The CP-knowers extend the cardinal principle to the whole counting list and display a proficient use of counting (Sarnecka & Carey, 2008). The achievement of the cardinality principle is an effortful process which engages children for approximately over two years, usually from the age of 2–2½ to 4½–5 (see Almoammer et al., 2013, for cross cultural variations due to linguistic structure).

Before entering the first year of formal schooling, children have also shown the ability to spatially map[2] Arabic numbers onto a visual line. In a seminal study, Siegler and Opfer (2003) asked children to place target numbers onto a horizontal line denoting a specific numerical interval (e.g., 0–100) by marking its position with a pencil. As demonstrated in many studies that employed this Number-to-Position task, hereafter Number Line (NL) task (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Siegler & Booth, 2004; Siegler & Lortie-Forgues, 2014; Siegler & Opfer, 2003), children initially overestimate small numbers and underestimate the position of larger numbers. Progressively, with age and greater familiarity with the numerical interval, children map numbers near the correct location thus showing a linear and accurate positioning. However, accuracy in positioning numbers on a small interval does not grant success on a larger interval. For instance, second graders display an accurate and linear mapping when placing numbers on the 0–100 interval but revert to a biased mapping when placing numbers on the 0–1000 interval (Siegler & Opfer, 2003). The shift from a biased to an accurate (and formally correct) mapping of numbers has been explained as: (a) the consequence of a shift from a logarithmic to linear representation of numbers (Siegler & Opfer, 2003); (b) the increasing ability and precision in performing a proportional judgement (Barth & Paladino, 2011; Slusser, Santiago, & Barth, 2013); (c) the increased knowledge and familiarity with both the proposed numbers and numerical intervals (Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008; Hurst, Leigh Monahan, Heller, & Cordes, 2014; Moeller, Pixner, Kaufmann, & Nuerk, 2009); and (d) the development of measurement skills (Cohen & Sarnecka, 2014). Regardless of the theoretical interpretation of this finding, the ability to accurately map numbers on the line is strongly correlated with performance in more complex numerical tasks, as well as with mathematical achievement and arithmetic proficiency (Booth & Siegler, 2008). Crucially, performance in the NL task is a powerful predictor of overall mathematics achievement, which remains reliable even after controlling for arithmetic, reading achievement, and IQ (Booth & Siegler, 2006, 2008; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). Moreover, children with math disability display a less accurate performance in the NL task as compared to typical developing children (Geary, Hoard, Nugent, & Byrd-Craven, 2008; Landerl, Bevan, & Butterworth, 2004; Sella, Berteletti, Brazzolotto, Lucangeli, & Zorzi, 2014). Finally, performance in the NL task is correlated with neural activation specific to arithmetical processing (Berteletti, Man, & Booth, 2015).

Though the NL task (and its variants) has been extensively used with different numerical and non-numerical intervals (Berteletti, Lucangeli, & Zorzi, 2012; Sella, Berteletti, Lucangeli, & Zorzi, 2015b; Siegler, Thompson, & Opfer, 2009), little is known about the acquisition of the spatial mapping for numbers at early stages of development. In the largest study on preschoolers to date, Berteletti et al. (2010) administered the NL task with three numerical intervals (i.e., 1–10, 1–20, 0–100) to pupils belonging to three different age groups (youngest group: $M_{age} \approx 4$ y.o.; middle group: $M_{age} \approx 5$ y.o.; oldest group: $M_{age} \approx 6$ y.o.). In the 1–10 interval, individual mapping analysis highlighted that children shifted from a biased (logarithmic) to an accurate (linear) mapping with age. Nevertheless, a consistent group of children displayed an inconsistent (i.e., not numerically meaningful) mapping throughout the age groups (52% in the youngest group; 38% in the middle age group; 15% in the oldest group). On average, children aged 5 years and above show a linear mapping for the 0/1–10 numerical interval (Berteletti et al., 2010; also see Muldoon, Towse, Simms, Perra, & Menzies, 2013; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; Sasanguie, Van den Bussche, & Reynvoet, 2012), whereas younger children either show a biased mapping or use a non-numerical strategy to place numbers (e.g., placing numbers in the middle of the line or alternating between right and left side answers irrespective of numerical value). Siegler and Ramani (2008) administered the NL task with the 1–10 interval to preschool children (between 4 and 5 years of age) as a pre-test measure in a training study testing the effectiveness of playing linear board games in enhancing linear numerical representation. In the pre-training analysis, children mainly displayed a non-numerical estimation strategy (e.g., placing numbers in the middle of the line) whereas, after actively playing with a linear board game, children's mapping became linear (see also Ramani, Siegler, & Hitti, 2012; Siegler & Ramani, 2009).

As the above studies demonstrate, counting and spatial mapping of numbers are two numerical abilities that emerge already in young preschool children. However, little is known about the relation between these two skills. Young, Marciani, and Opfer (2011) found a positive correlation between linearity in the NL task with the 0–20 interval and performance in the GaN task in children between 3 and 5 years of age. Muldoon et al. (2013) also found a correlation between linearity in different numerical intervals (i.e., 0–10, 0–20, 0–100) and counting abilities in a sample of preschool children. Moreover, it has been found that spatial-numerical training enhances children's performance both in the NL task and in counting (Fischer, Moeller, Bientzle, Cress, & Nuerk, 2011). Berteletti et al. (2010) suggested that the acquisition of the cardinality principle plays a central role in spatial mapping because the ability of preschool children to map numbers linearly was correlated with the ability to numerically order sets of dots. Moreover, in a subsequent study, Berteletti et al. (2012) suggested that linearity is acquired in the numerical domain first and then generalized to other non-numerical ordinal sequences because only numbers

[2] An encyclopaedic definition of mapping is that of "operation that associates each element of a given set with one or more elements of a second set" (e.g., Stevenson & Lindberg, 2010). In the present paper, we use the term spatial mapping to refer to the association between each number and a spatial position on the visual line (for a more general definition of number-space mapping, see Nunez & Fias, 2015).