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Evaluating dimensional distinctness with correlated-factor models: Limitations and suggestions☆

Gilles E. Gignac[a,*], André Kretzschmar[b]

[a] School of Psychology, University of Western Australia, Australia
[b] Hector Research Institute of Education Sciences and Psychology, Eberhard Karls Universität Tübingen, Germany

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ABSTRACT

Many differential cognitive psychologists appear to interpret correlated-factor models associated with inter-latent variable correlations meaningfully < 1.0 as support for the plausibility of several related, but to some degree distinct, dimensions. It is contended in this paper that such a conclusion drawn from a well-fitting correlated-factor model may not be justified necessarily, even if the correlated-factor model fits better than a single-factor model. Based on a series of simulated correlation matrices, it is demonstrated that a well-fitting correlated-factor model with inter-latent variable correlations < 1.0 only suggests the possibility of several distinct group-level dimensions. In order to obtain additional useful information relevant to the distinctness of the hypothesized group-level factors, a higher-order model is demonstrated to be particularly useful, especially when complemented with omega hierarchical subscale (omegaHS), an effect size index of unique latent variable strength. The following guidelines are provided to help interpret the magnitude of omegaHS values: relatively small < 0.20; typical 0.20 to 0.30; and relatively large > 0.30. The implications of the simulation are demonstrated based on the re-analysis of three previously published correlations relevant to cognitive processes. Researchers are encouraged to supplement correlated-factor model analyses with higher-order model analyses, in order to evaluate the distinctness of the hypothesized dimensions more fully.

1. Introduction

Researchers have been recommended to use a competing models strategy in the application of confirmatory factor analysis (Jöreskog, 1993). A commonly observed and recommended model comparison is that between a single-factor model and a correlated-factor model (Brown, 2015; Byrne, 2010; Kline, 2011; Zeller & Carmines, 1980). In practice, differential cognitive psychologists who endorse a correlated-factor model acknowledge the association between the latent variables. However, they also tend to make statements relevant to the separability, or uniqueness, of each of the hypothesized specific dimensions. For example, McAuley and White (2011) reported a well-fitting correlated three-factor model with inter-latent variable correlations < 1.0 and concluded that: "This model is consistent with the hypothesis that processing speed, response inhibition, and working memory are separable abilities" (p. 461). In another investigation, Neubert, Kretzschmar, Wüstenberg, and Greiff (2015) endorsed a well-fitting correlated-factor model from the perspective that the latent variables represented "…strongly related, but nonetheless separable dimensions of CPS [complex problem solving] …" (p. 186). Finally, Miyake et al. (2000) stated that the "… three target functions (i.e., shifting, updating, and inhibition) are clearly distinguishable … [but] do seem to share some underlying commonality" (pp. 86–87), based on a well-fitting correlated three-factor model with inter-latent variable correlations < 1.0. There are many similar examples in the cognitive literature (e.g., Alloway, Gathercole, & Pickering, 2006; Deary, McCrimmon, & Bradshaw, 1997; Engel de Abreu, Conway, & Gathercole, 2010; Friedman et al., 2006; Giofrè, Mammarella, & Cornoldi, 2013; Greiff et al., 2013; Hegarty, 2004; Hicks, Harrison, & Engle, 2015; Janssen, De Boeck, & Vander Steene, 1996; Kail & Hall, 2001; Mackintosh & Bennett, 2003; Miyake, Friedman, Rettinger, Shah, & Hegarty, 2001; Santos et al., 2015; Shelton, Elliott, Matthews, Hill, & Gouvier, 2010; Unsworth, Spillers, & Brewer, 2009). In our view, such statements about evidence in favour of the interpretation of specific, unique dimensions are ambiguous, if not unjustified, without additional testing.

In the first part this paper, it will be demonstrated via simulation that a well-fitting correlated three-factor model that fits better than a single-factor model is not necessarily indicative of the plausibility of three or more unique, group-level dimensions, even when the inter-latent variable correlations are substantially < 1.0. Instead, in order to

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* Corresponding author at: School of Psychology, University of Western Australia, 35 Stirling Highway, Crawley, Western Australia 6009, Australia.
E-mail address: gilles.gignac@uwa.edu.au (G.E. Gignac).

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evaluate clearly the proposition that there are a specified number of specific, group-level dimensions within the data, researchers are encouraged to conduct supplementary model testing; specifically, the higher-order model with emphasis placed on the first-order factor residuals, in addition to omega hierarchical subscale estimates (omegaHS; ω_{hs}).

In the second part this paper, the limitations associated with the correlated-factor model will be demonstrated based on the re-analysis of correlation matrices reported in three previously published confirmatory factor analytic investigations in the area of differential cognitive psychology. To foreshadow the results, it will be shown that a well-fitting correlated-factor model with inter-latent variable correlations < 1.0 may, or may not, be associated with as many unique group-level dimensions as suggested by the authors. Consequently, it will be argued in this paper that the results associated with a correlated-factor model should not be relied upon, on their own, for the purposes of evaluating what appears to be a typically observed interpretation of a correlated-factor model in the literature.

2. Single-factor versus correlated-factor models: described

The single-factor model is one of simplest models that can be specified to account for the shared variance between indicators (Kline, 2011; Rindskopf & Rose, 1988; Spearman, 1904). As can be seen in Fig. 1, Model 1 depicts a single-factor model with one latent variable defined by nine indicators. Theoretically, the single-factor model implies one general construct (g), the presence of which, theoretically, causes the indicators to inter-correlate with each other. In this case, the single-factor model is associated with 27 degrees of freedom.

Model 2 is an example of a restricted correlated three-factor model. There are three latent variables, A, B and C, each defined by three indicators. Furthermore, there is a covariance term between all three latent variables to represent their inter-association. The correlated-factor model is a more complex model than the single-factor model (Rindskopf & Rose, 1988). Correspondingly, in this case, the restricted correlated three-factor model is associated with three fewer degrees of freedom (i.e., 24). Theoretically, the variance common to all indicators in a correlated three-factor model is implied to be due to the shared variance between the three latent variables (a.k.a., group-level factors), rather than a general or global process. The single-factor model and the correlated three-factor model are nested within each other, as the correlated-factor model can recover a single-factor model with three fixed parameters, i.e., the inter-latent variable correlations fixed to 1.0.

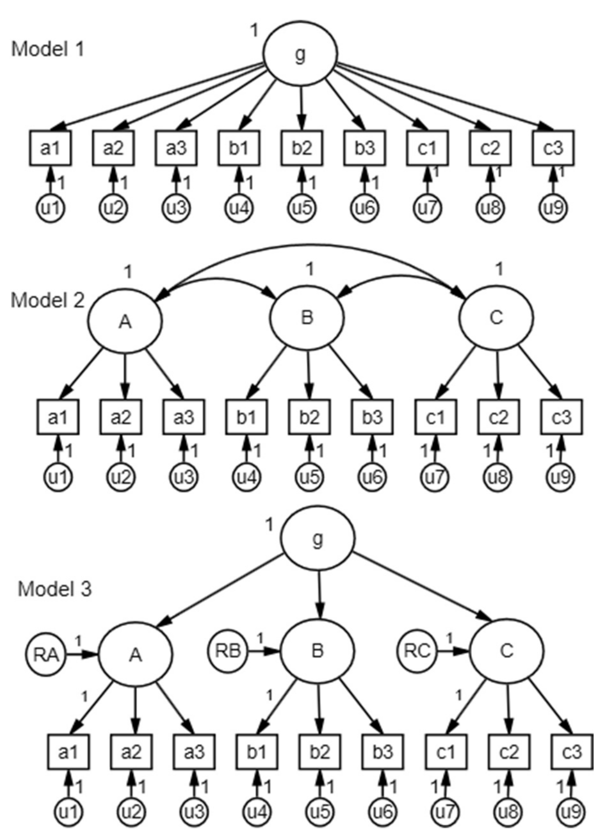

Fig. 1. Competing measurement models; Model 1 = single-factor model; Model 2 = correlated factor model; Model 3 = higher-order model.

In practice, some differential cognitive psychologists appear to interpret well-fitting correlated-factor models, with inter-latent variable correlations < 1.0, from the perspective that the latent variables represent, to some degree, distinct or specific dimensions (e.g., Adrover-Roig, Sesé, Barceló, & Palmer, 2012; Alloway et al., 2006; Deary et al., 1997; Engel de Abreu et al., 2010; Friedman et al., 2006; Giofrè et al., 2013; Greiff et al., 2013; Hegarty, 2004; Hicks et al., 2015; Janssen et al., 1996; Kail & Hall, 2001; Mackintosh & Bennett, 2003; McAuley & White, 2011; Miyake et al., 2001; Santos et al., 2015; Shelton et al., 2010; Swanson, Orosco, & Lussier, 2015; Unsworth et al., 2009). However, in order to support the notion that there are three or more distinct dimensions within a correlated-factor model, it is suggested in this paper that there should be some unique (unshared) true score variance associated with each of the postulated group-level latent variables/dimensions. In practice, it is difficult to determine whether there is, in fact, unique true score variance associated with each of the latent variables specified within a correlated-factor model solution. As will be demonstrated in the simulation below, it is possible that less than all three of the latent variables (A, B, C) may be found to be associated with clear empirical support (i.e., unique true score variance), even when the inter-latent variable correlations between the latent variables are substantially < 1.0. To overcome the limitations of the correlated-factor model, it is argued in this paper that the higher-order model (Burt, 1950; Rindskopf & Rose, 1988; Thomson, 1951) can be especially useful for the purposes of determining, relatively unambiguously, whether all of the postulated distinct dimensions associated with a correlated-factor model are, in fact, clearly empirically supported representations of distinct or separable dimensions.

As can be seen in Fig. 1 (Model 3), the higher-order model is associated with one general factor defined by three first-order factors (A, B, and C). Often unrecognized is that the higher-order model is, typically, associated with orthogonal latent variable terms (Gignac, 2016). In the current example, there are three orthogonal latent variable terms. Specifically, all of the first-order factors are associated with a residual: RA, RB, and RC. The RA, RB, and RC terms represent the true score variance (i.e., not error variance) that was unaccounted for by the general factor. For example, if the A first-order factor were defined by operation span, an *n*-back task, and letter-number sequencing, the A first-order factor residual (RA) would represent the variance common to the three tasks that was not shared with the other indicators in the model. Thus, depending on the nature of the other indicators in the model, the RA term may represent an independent (or residualised of *g*) working memory capacity construct. By contrast, the general factor may be considered a global memory span dimension. For the purposes of the argument advanced in this manuscript, it is not necessary to postulate theoretically the presence of a general or global dimension of any nature. Instead, the general or global factor is simply specified to help estimate the amount of unique variance associated with the hypothesized group-level dimensions. Thus, specific information from a higher-order model is encouraged to be used simply to evaluate a possibly theoretically preferred correlated-factor model.

As a first-order factor's loading onto the general factor increases, the amount of variance associated with a first-order factor's residual will decrease. It is the position of this paper that should a statistically significant residual variance term be observed, then there would be clear evidence to suggest that a particular dimension is, a least partly,