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# Bayesian statistical reconstruction for low-dose X-ray computed tomography using an adaptive-weighting nonlocal prior☆

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ABSTRACT

How to reduce the radiation dose delivered to the patients has always been a important concern since the introduction of computed tomography (CT). Though clinically desired, low-dose CT images can be severely degraded by the excessive quantum noise under extremely low X-ray dose circumstances. Bayesian statistical reconstructions outperform the traditional filtered back-projection (FBP) reconstructions by accurately expressing the system models of physical effects and the statistical character of the measurement data. This work aims to improve the image quality of low-dose CT images using a novel *AW nonlocal* (*adaptive-weighting nonlocal*) prior statistical reconstruction approach. Compared to traditional *local* priors, the proposed prior can adaptively and selectively exploit the global image information. It imposes an effective resolution-preserving and noise-removing regularization for reconstructions. Experimentation validates that the reconstructions using the proposed prior have excellent performance for X-ray CT with low-dose scans.



## 1. Introduction

Since the introduction of computed tomography (CT) in the 1970s, the X-ray exposure represents the largest source of radiation exposure because of the rapidly increasing use of X-ray CT. Minimizing the radiation exposure to patients has been one of the major concerns in the CT field and modern clinical radiology [1]. On the other hand, HELICAL/SPIRAL CT (HCT) makes it possible to scan adequately a vital organ volume in a single breath-hold, thus problems related to patient motion can be minimized. In recent years, HCT has replaced conventional stop-and-shoot CT in many clinical applications. The helical scanning is of significant benefit to CT angiography and multi-phase abdominal imaging. Although HCT has many advantages, the effective dose of HCT can be up to four times higher than that of a corresponding conventional stop-and-shoot CT [1,2].

Low-dose CT imaging has been under development in the last decade and is currently clinically desired [2,3]. A simple and cost-effective means among many strategies to achieve low-dose CT applications is to lower X-ray tube current (mA) as low as achievable (for both helical and stop-and-shoot modes). However, the image quality of low mA acquisition protocol will be severely degraded due to the excessive X-ray quantum noise. Statistical iterative reconstruction approaches have shown good results in application and usually outperform the FBP method as the noise increases [4–13]. Statistical methods can also incorporate the object constraints and prior information into the reconstruction through Bayesian approaches. Based on Markov random fields (MRF) theory [13], the original ill-posed reconstruction can be greatly improved by Bayesian methods [8–12].

How to devise an effective prior for Bayesian reconstruction has been widely studied in the past 10 years [8–14]. Lowering the noise effect and preserving the edge are the two main aims in devising priors. Under Bayesian and MRF paradigm, the work in this paper is to devise an effective prior for low-dose X-ray statistical tomographic reconstruction.

The simple and widely used quadratic membrane (QM) prior tends to produce an unfavorable oversmoothing effect. And some edge-preserving nonquadratic priors are able to produce sharp edges by choosing a nonquadratic prior energy [8–13]. And in 1998, edge-preserving median root prior (MRP) was proposed by Alenius and his colleagues for iterative reconstruction of PET transmission images [11]. However, in the case of low X-ray scan when the noise level is relatively significant, such edge-preserving nonquadratic

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priors tend to produce blocky piecewise regions or staircase artifacts. None of these priors addresses the information of global connectivity and continuity in objective image. Only local and indiscriminatively prior information is provided. We term these traditional priors *local* priors.

Yu and Fessler devised a boundary-based Bayesian method which incorporates global information of image by level-set methods [12]. But such boundary-based method relies heavily on the level-set operations whose effect in different images is unpredictable and parameter-dependent. Recently, Buades et al. put forward a novel algorithm for image denoising [14]. Illuminated by their nonlocal idea, a nonlocal MRF quadratic prior model for Bayesian image reconstruction is proposed [15]. In this article, we propose a novel *AW nonlocal* (*adaptive-weighting nonlocal*) prior for X-ray CT Bayesian reconstruction. Such AW nonlocal prior reconstruction approach can selectively and adaptively include the relevant neighbor pixels for the regularization,and eliminate the negative regularization effect from the irrelevant neighbor pixels. In Section 2, a review of the old prior model and the theory for the proposed AW nonlocal prior model are both illustrated. In Sections 3 and 4, we give the iterative reconstruction algorithm and perform simulated statistical CT image reconstruction with two different dose levels. Relevant comparisons show the proposed AW nonlocal prior's good properties in low-dose X-ray computed tomography. Conclusions and plans for future work are given in Section 5.

## 2. Theory of the proposed aw nonlocal prior model

Based on Bayesian and MRF theory, regularization from prior information can be imposed on image reconstruction to suppress noise effect. And we can build following posterior probability $P(f/g)$ for image reconstruction.

$$P\left(\frac{f}{g}\right) \propto P\left(\frac{g}{f}\right) P(f) \tag{1}$$

$$P(f) = Z^{-1}\exp(-\beta U(f)) = Z^{-1}\exp(-\beta \sum_j U_j(f)) \tag{2}$$

where $P(g/f)$ is the likelihood distribution. $P(f)$ is the prior distribution. The partition function $Z$ is a normalizing constant. $U(f)$ is the prior energy function, and $U_j(f)$ is the notation for the value of the energy function $U$ evaluated on the $f$ at pixel index $j$. $\beta$ is the global hyperparameter that controls the degree of the prior's influence on the image $f$. The energy function in (2) attains its minimum and the corresponding prior distribution (2) attains its maximum when the image meets the prior assumptions ideally. And we can build the posterior energy function as

$$\psi_\beta\left(\frac{f}{g}\right) = \log P\left(\frac{f}{g}\right) = L(g,f) - \beta U(f) \tag{3}$$

where $L(g,f)$ represents the likelihood energy function. The reconstructed image $f$ can be obtained through maximization of function $\psi_\beta(f)$ by an iterative procedure.

### 2.1. The traditional local prior model

According to Bayesian theory, when the image $f$ meets the prior assumptions, the prior energy function $U(f)$ in (3) attains its minimum and the corresponding prior distribution (3) attains its maximum. Conventionally, the value of $U_j(f)$ is computed through a weighted sum of potential functions $v$ of the differences between pixels in the neighborhood $N_j$:

$$U(f,j) = \sum_{b \in N_j} w_{bj} v(f_b - f_j) \tag{4}$$

Generally, different choices of the potential function $v$ lead to different priors. The prior become the space-invariant QM prior with the potential function $v(t) = t^2/2$.

We can also choose edge-preserving nonquadratic priors by adopting a nonquadratic potential function for $v$, such as the Huber potential function:

$$v(t) = \begin{cases} t^2/2, & |t| \le \gamma \\ \gamma|t| - \gamma^2/2, & |t| > \gamma \end{cases} \tag{5}$$

where $\gamma$ is the threshold parameter [9,13].

Such edge-preserving nonquadratic prior preserves the edge information by choosing a nonquadratic potential function that increases less as the differences between adjacent pixels become bigger [9,13]. And in iterative reconstruction algorithm using MRP, the medians of the pixels within local neighborhoods are incorporated into the iterative algorithm to remove noise without blurring edges [11].

Weight $w_{bj}$ is a positive value that denotes the interaction degree between pixel $b$ and pixel $j$. And in local prior model, it is usually considered to be inversely proportional to the distance between pixel $b$ and pixel $j$. So on a square lattice of image $f$, in which the 2D positions of the pixel $b$ and pixel $j$ ($b \neq j$) are respectively $(b_x, b_y)$ and $(j_x, j_y)$, $w_{bj}$ is usually calculated by the formula $1/\sqrt{(b_x - j_x)^2 + (b_y - j_y)^2}$ or some other simple forms. Typical normalized eight-neighborhood and four-neighborhood weighting maps for local priors are depicted as follows:

$$1,\quad \left(1\Big/\left(4\times 1 + 4\times 1\big/\sqrt{2}\right)\right) \times \boxed{\begin{matrix} 1/\sqrt{2} & 1 & 1/\sqrt{2} \\ 1 & 0 & 1 \\ 1/\sqrt{2} & 1 & 1/\sqrt{2} \end{matrix}}$$

1.

$$2,\quad \left(1/(4\times 1)\right) \times \boxed{\begin{matrix} & 1 & \\ 1 & 0 & 1 \\ & 1 & \end{matrix}}$$

2.

### 2.2. The proposed AW nonlocal prior model

A novel *nonlocal prior* is devised for Bayesian image reconstruction in [15]. Based on the nonlocal prior, we propose a new *AW nonlocal* (*adaptive-weighting nonlocal*) prior for X-ray computed tomography. A large neighborhood $N$ is also set to incorporate geometrical configuration information in the image. The weight $w_{bj}$ for the new selective nonlocal prior is set to be an 1-0 binary function. The value of $w_{bj}$ is 0 or 1 when the distance $d_{bj}$ is greater or smaller than a set threshold $\delta$. The value of the distance $d_{bj}$ is determined by a distance measurement between $n_b$ and $n_j$ centered at pixel $b$ and pixel $j$, respectively. Thus only the relevant ones are picked out for the AW nonlocal prior. The two neighborhoods $n_b$ and $n_j$ have the same size. The building of the proposed AW nonlocal prior can be formalized as follows:

$$U_{AW\_NL}(f) = \sum_j U_j(f) = \sum_j \sum_{b \in N_j} w_{bj}(f_b - f_j)^2 \tag{6}$$

$$w_{bj} = \begin{cases} 1 & d_{bj} \le \delta \\ 0 & d_{bj} > \delta \end{cases} \tag{7}$$

$$d_{bj} = \left| f(n_b) - f(n_j) \right|_E^2 = \sum_l (f_{l \in n_b} - f_{l \in n_j})^2 \tag{8}$$

$$f(n_b) = \{f_l : l \in n_b\} \tag{9}$$