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X-ray angiogram images enhancement by facet-based adaptive anisotropic diffusion

Guodong Wang, Nong Sang, Luxin Yan*, Xubang Shen

Institute for Pattern Recognition and Artificial Intelligence, Huazhong University of Science and Technology, Wuhan, 430074 Hubei, China

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ABSTRACT

The paper presents a versatile nonlinear diffusion method to visually enhance the angiogram images for improving the clinical diagnosis. Traditional nonlinear diffusion has been shown very effective in edge-preserved smoothing of images. However, the existing nonlinear diffusion models suffer several drawbacks: sensitivity to the choice of the conductance parameter, limited range of edge enhancement, and the sensitivity to the selection of evolution time. The new anisotropic diffusion we proposed is based on facet model which can solve the issues mentioned above adaptively according to the image content. This method uses facet model for fitting the image to reduce noise, and uses the sum square of eigenvalues of Hessian as the standard of the conductance parameter selection synchronously. The capability of dealing with noise and conductance parameter can also change adaptively in the whole diffusion process. Moreover, our method is not sensitive to the choice of evolution time. Experimental results show that our new method is more effective than the original anisotropic diffusion.

1. Introduction

X-ray angiogram image plays a very important role in clinical diagnosis. However, the image quality is usually not very good owing to various reasons such as the complex background and some noise. So it is necessary to enhance these images to facilitate the following processing or diagnosis. Much effort has been spent on this problem with a focus on multi-scale filtering [18,21–24]. These methods were proved to be effective to the images with little noise [21–23], while to those with large noise, the edges might be destroyed because the Gaussian filter was introduced to their implementations [18,24].

The anisotropic diffusion proposed by Perona and Malik [2] is a powerful tool to solve the problem met in angiogram enhancement. Anisotropic diffusion methods were popularly used in image processing in the past decade [1–3,8,11,12]. The feature of these functions was reflected by their diffusion coefficients. The most well-known diffusion coefficients were proposed by Perona and Malik [2], but Catte et al. had shown that they led to an ill-posed diffusion: a small perturbation in the data might cause significant changes in the result [3,4]. To deal with this problem, they used a Gaussian-convolved version [5] of noisy images to compute diffusion coefficients. However, the method smoothed the edges in images due to the linear filter, although it could rapidly eliminate noise at the same time. Furthermore, how to select a proper Gaussian standard deviation was another serious problem. Generally, on the one hand, the smaller the standard deviation was, the poorer the effect of smoothing was. On the other hand, a large standard deviation would make the smoothing overrunning so that the smoothed images would be blurred. Besides, during the procedure the noise became little and little so that it was not suitable to use an unchanged standard deviation.

Another problem in fixation of diffusion coefficients is how to select a proper conductance parameter. Traditional conductance parameter is fixed and consistent in the whole image. Li [13] proposed to make the conductance parameter adaptive according to the content in angiogram images. But in the process of iteration the conductance parameter is unchanged in spite of the altered image content. The two questions above were also mentioned in [14]; Li used special forms of $\sigma(t)$ and $K(t)$. Although his experimental result showed it could get a better result, the selection of $\sigma(t)$ and $K(t)$ also needed experimental decision.

In this paper, a new adaptive anisotropic diffusion based on facet model for angiogram images [6,9,10] is introduced. The facet model is a powerful tool for image processing; it is used in many aspects [6,7,9,10]. By using facet model the questions mentioned above can be adaptively solved in angiogram images.

2. Anisotropic diffusion models

Before introducing the anisotropic diffusion, we define some notations first. I represents an image and $I(t, x, y)$ the gray value

* Corresponding author.
E-mail address: yanluxin@gmail.com (L. Yan).

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of position (x, y) in image I at time t; its gradient is given by $\nabla I = [I_x, I_y]^T$ and the magnitude of its gradient by $|\nabla I|$. The divergence of a vector $\vec{E}$ is denoted as $div(\vec{E})$.

The basic anisotropic diffusion equation is

$$\frac{\partial I}{\partial t} = div[c(\left|\nabla I\right|)\nabla I] \tag{1}$$

where $c(x)$ is the anisotropic diffusion coefficient. A desirable behavior of this function is that Eq. (1) diffuses more in smooth areas and less across edges. The following equation gives one of the functions with such a quality:

$$c(\left|\nabla I\right|) = \frac{1}{1 + (\left|\nabla I\right|/K)^2} \tag{2}$$

where K is the conductance parameter. To deal with the ill-posed diffusion due to noise, Catte et al. use

$$\frac{\partial I}{\partial t} = div[c(\left|I * G_\sigma\right|)\nabla I] \tag{3}$$

where $I * G_\sigma$ is the convolution of I and a Gaussian filter with standard deviation σ. In Eq. (3), σ is fixed, which seems not quite desirable because if the unwanted intensity variations diminish more rapidly than the signal of interest, the gradient measurements will become more and more reliable so that the uniform smoothing becomes less important as the equation evolves. This suggests that the scale parameter σ should be decreased as time goes by. So how to choose a proper $\sigma(t)$ to achieve optimal results is still a considerable problem. Different $\sigma(t)$ would give different results. With the evolution of the processing, more and more noise will be smoothed, so the low-pass filtering performance introduced for diminishing noise in each iterative step should become weaker and weaker.

Another problem of implementing anisotropic diffusion is how to select the conductance parameter. The value of the conductance parameter K greatly affects the diffusion. Generally speaking, a large K will lead to smoothness whereas a little K will lead to sharpness in local regions of images. A fixed K during the diffusion process will simplify the implementation. However, a problem will be caused if the value of K is not suitable, i.e. the region we want to enhance will be smoothed instead. On the contrary, if the value of K is too low, it will cause enhancement where there is much noise. Furthermore, a too low K will weaker the diffusion effects, which results in too many iterations. In this case the parameter should be equilibrated. Moreover, different images have different ideal values of K. Therefore, a natural approach to solving the problems above is to make this parameter adaptive.

In [14], Li pointed out that conductance parameter K should change during the whole process of diffusion. He gave some typical forms of $K(t)$ which couldn't be applied for all images perfectly.

3. Facet-model based anisotropic diffusion

To solve the problems mentioned above, we use facet model to perfect the anisotropic diffusion. Our method mainly consists of two aspects. One is use of facet model fitting to deal with noise adaptively and the other is using Hessian matrix to assist decision of the conductance parameter according to the image. Thus our proposed method can deal with different images adaptively.

3.1. Introduction of cubic facet model

The cubic facet model [6,9,10] assumes that in each neighborhood of an image, the underlying gray-level intensity surface can be approximated by a bivariate cubic function f. The two-dimensional discrete orthogonal polynomial (DOP) basis set can be constructed from the tensor product of the two sets of one-dimensional discrete polynomials. For a cubic function, the polynomial bases with order higher than 3 can be ignored.

Let S be a symmetric 2D neighborhood defined on $R \times C$, and $I(r, C)$ be the observed intensity value at $(r, c) \in S$. Let $\{g_0(r, c), g_1(r, c), \ldots, g_N(r, c)\}$ be the set of 2D DOP basis functions. As a result, for instance with 5×5 pixels kernel size, the bivariate cubic function $f(r, c)$, expressed using discrete orthogonal polynomials, is

$$\begin{aligned} f(r,\ c) = {} & K_1 + K_2 r + K_3 c + K_4(r^2 - 2) + K_5 rc + K_6(c^2 - 2) \\ & + K_7\left(r^3 - \frac{17}{5}r\right) + K_8(r^2 - 2)c + K_9 r(c^2 - 2) \\ & + K_{10}\left(c^3 - \frac{17}{5}c\right) \end{aligned} \tag{4}$$

where K_i, $i = 1, \ldots, 10$ are coefficients for the bivariate cubic function expressed in discrete orthogonal polynomials.

In the cubic facet model, each facet centered about a given pixel may be approximated by the bivariate cubic function in canonical form, as shown in (4). Evaluating the first row and column partial derivatives at the neighborhood centre (0, 0) (i.e. $r = 0$ and $c = 0$) yields the first and second directional derivatives.

$$\frac{\partial f}{\partial r} = K_2 - \frac{17}{5}K_7 - 2K_9, \qquad \frac{\partial f}{\partial c} = K_3 - \frac{17}{5}K_{10} - 2K_8,$$
$$\frac{\partial^2 f}{\partial r^2} = 2K_4, \qquad \frac{\partial^2 f}{\partial r \partial c} = K_5, \qquad \frac{\partial^2 f}{\partial r \partial c} = K_5, \qquad \frac{\partial^2 f}{\partial c^2} = 2K_6 \tag{5}$$

where K_i are the fitting coefficients. Each coefficient K_i can be computed independently by convolving the image with the corresponding weight kernel. For more details about cubic facet model fitting, please see Refs. [6,9,10].

3.2. The conductance parameter as a function of evolution

For angiogram images, Li [13] used Hessian matrix to assist decision of the conductance parameter K according to the image. To achieve this goal, we turn to make a local analysis of the image. One common way is to perform Taylor expansion in the neighborhood of a point x:

$$I(x + \nabla x) \approx I(x) + \nabla x^T\ \nabla I(x) + \Delta x^T H(x)\,\Delta x \tag{6}$$

where ∇x and $H(x)$ are the gradient and the Hessian matrix at time t and at the point x, respectively. The Hessian matrix at one point in the image is

$$H = \begin{bmatrix} \dfrac{\partial^2 I}{\partial x^2} & \dfrac{\partial^2 I}{\partial x \partial y} \\ \dfrac{\partial^2 I}{\partial y \partial x} & \dfrac{\partial^2 I}{\partial y^2} \end{bmatrix} \tag{7}$$

We assume that the Hessian matrix at one point has two eigenvalues λ_1 and λ_2. From experiments we get that the square sum of eigenvalues is small in vein regions and large in backgrounds. Considering the fact that we are more interested in regions containing vessels in angiogram images, we hope to achieve enhancement in these regions. On the contrary, in the regions where the measures are high we hope to achieve smoothing since they are more likely to be the background. That is to say, the conductance parameter should be high in the regions where the measures are high and be low in the regions where the measures are low. So the conductance parameter should be different in each pixel in image.

Based on the discussion above, we contrive the conductance parameter as a function of the model as follows:

$$K = (\lambda_1^2 + \lambda_2^2)^{1/2} \tag{8}$$