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Image formation in vibro-acoustography with depth-of-field effects ☆

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Abstract

We study the image formation of vibro-acoustography systems based on a concave sector array transducer taking into account depth-of-field effects. The system point-spread function (PSF) is defined in terms of the acoustic emission of a point-target in response to the dynamic radiation stress of ultrasound. The PSF on the focal plane and the axis of the transducer are presented. To extend the obtained PSF to the 3D-space, we assume it is a separable function in the axial direction and the focal plane of the transducer. In this model, an image is formed through the 3D convolution of the PSF with an object function. Experimental vibro-acoustography images of a breast phantom with lesion-like inclusions were compared with simulated images. Results show that the experimental images are in good agreement with the proposed model.

Keywords: Vibro-acoustography; Point-spread function; Image formation

1. Introduction

Imaging elastic properties of tissues has become a new field in image sciences known as elastography. Variation in elasticity of soft tissues is commonly related to pathologies. Thereby, non-invasive techniques to image elasticity parameters of *in vivo* tissues are promising tools in medical diagnosis. The underlying principle of elastography is to measure or track the induced motion of tissue due to an external applied force [1]. One of the most effective ways to produce such force is to use radiation stress of ultrasound.

Vibro-acoustography is an elastography technique that images the response of an object (or tissue) to the low-frequency dynamic radiation stress produced by a localized ultrasound beam [2]. The dynamic radiation is caused by the wave momentum transferring to an object or propagating medium [3]. In vibro-acoustography, this stress is yielded by two co-focused monochromatic ultrasound beams of slightly different frequencies (typically in the kilohertz range). The tissue within the system focal zone is subjected to deformation/vibration at the beat frequency of the ultrasound beams. It, thus, irradiates a field (acoustic emission) which can be probed by an acoustic detector. The image is formed by pixels whose brightness is determined by the acoustic emission of each point in the tissue. The acoustic emission carries information of the tissue region at low and ultrasound frequencies.

The information at the ultrasound frequency is related to the medium scattering properties, as variation of density and compressibility. The low-frequency information may reveal elastic properties of tissue. Conventional ultrasound imaging does not provide this kind of information. Potential clinical applications of vibro-acoustography include imaging of lesions in soft tissues such as calcification [4,5], liver tumors [6], and following brachytherapy seeds [7].

The development of vibro-acoustography for medical imaging applications demands a detailed assessment of the image formation in this method. Under very sensible conditions, vibro-acoustography image formation can be described by the system point-spread function (PSF), which is related to the radiation stress field exerted on a point-target [8]. Thus, it is necessary to study the stress field forming by acoustic transducers. Stress field forming in vibro-acoustography has been study theoretically and experimentally for a two-element confocal and *x*-focal transducers [9], and an eight-element sector array transducer

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[10]. In these studies, the spatial resolution and sidelobe levels were obtained and validated experimentally. A study based on computational simulations for linear arrays can be found in ref. [11]. Despite those investigation, no much attention has been given to the effects of the PSF depth-of-field in the system image formation.

In this paper, we investigate the image formation in vibro-acoustography including depth-of-field effects. To achieve this, we propose a model in which the 3D PSF can be represented as a separable function in the axial direction and the focal plane of the transducer. Images can be simulated by performing the spatial convolution of the 3D PSF and an object described by means of a function. An experimental vibro-acoustography system acquired images of a breast phantom with lesion-like inclusions. Both experimental and simulated images are compared. The obtained results are in good agreement with the proposed image formation model.

2. Image formation theory

2.1. Dynamic radiation stress

In vibro-acoustography, the dynamic radiation stress “taps” an object embedded in the propagating medium; which, in response, emits an acoustic field (acoustic emission). The acoustic emission depends on the object’s shape and mechanical properties and is proportional to the dynamic radiation stress. The information carried by the acoustic emission is used to synthesize an image of the object. A vibro-acoustography imaging, as a linear system, can be characterized by its PSF. The vibro-acoustography PSF depends on the acoustic emission of a point-target. This target is considered here as a small sphere of radius a.

The dynamic radiation stress is produced by a transducer with two ultrasound sources. Each source is driven by a sinusoidal signal. The driving angular frequencies of the sources are $\omega_1 = \omega_0 + \Delta\omega/2$ and $\omega_2 = \omega_0 - \Delta\omega/2$, where ω_0 and $\Delta\omega$ are the center and difference frequencies, and $\Delta\omega \ll \omega_0$. The two ultrasound beams are focused at the same point in space producing a dynamic radiation stress at the beat frequency $\Delta\omega$. We consider an small sphere immersed in an infinitely extended homogeneous lossless fluid with density ρ_0 and speed of sound c_0. In this case ultrasound waves can be fully described in terms of the velocity potential $\phi(\mathbf{r}, t)$, where $\mathbf{r}$ is the position vector and t is the time.

The two interacting beams are supposed to behave like a bichromatic plane wave in the vicinity of the system focal zone. Having $\Delta\omega/\omega_0 \ll 1$, the complex amplitude of the radiation stress at $\Delta\omega$ exerted on the small sphere is, according to ref. [3], given by

$$\hat{\sigma} = k_0^2 \hat{y} \hat{\phi}_1 \hat{\phi}_2^*, \tag{1}$$

where $k_0 = \omega_0/c_0$ and $\hat{y}$ is the complex radiation force function of the sphere which depends on the scattering properties of the sphere mostly at the ultrasound frequency ω_0. The functions $\hat{\phi}_1$ and $\hat{\phi}_2$ are the complex amplitude of the velocity potentials of the ultrasound beams.

2.2. Point-spread function

We are now interested in obtaining the acoustic emission by a point-target modelled as an small rigid sphere. The sphere is supposed to oscillate in linear regime with small amplitude around the coordinate system origin along the z axis. The sphere vibration velocity in the steady-state is $\hat{v}e^{j\Delta\omega t}$, in which $\hat{v}$ is the complex velocity amplitude. The sphere radius is much smaller than the wavelength of the incident ultrasound waves. Hence, $\Delta ka \ll 1$, where $\Delta k = \Delta\omega/c_0$ is the wavenumber of the acoustic emission. The amplitude of the emitted pressure by the oscillating sphere is, according to ref. [12], a dipole radiation given, in spherical coordinates (r, θ, φ) by

$$\hat{p} = \rho_0 c_0 \hat{v} \Delta k^3 a^3 \cos\theta \frac{e^{-j\Delta kr}}{2\Delta kr}. \tag{2}$$

The velocity amplitude $\hat{v}$ of sphere can described by its mechanical impedance $\hat{z}$ at the frequency $\Delta\omega$ as follows $\hat{v} = \pi a^2 \hat{\sigma}/\hat{z}$. The acoustic outflow (the volume of the medium which is displaced per unit time due to an object vibration per unit force) by the sphere is related to the mechanical impedance by the relation $\hat{q}(\Delta\omega) = 2\pi a^2/\hat{z}(\Delta\omega)$. Thus, Eq. (2) may be rewritten as

$$\hat{p} = \rho_0 c_0 \hat{\sigma}(\omega_0) \hat{q}(\Delta\omega) \hat{g}(r, \theta), \tag{3}$$

where

$$\hat{g}(r, \theta) = \Delta k^3 a^3 \cos\theta \frac{e^{-j\Delta kr}}{\Delta kr}$$

is the a transfer function which depends on the medium and the object boundary-conditions. By using Eqs. (1) and (3), we may write the acoustic emission amplitude of an sphere located at the position $\mathbf{r}$ and measured at $\mathbf{r}'$ as follows

$$\hat{p}(\mathbf{r}'|\mathbf{r}) = \rho_0 c_0 k_0^2 \hat{y}(\omega_0) \hat{q}(\Delta\omega) \hat{\phi}_1(\mathbf{r}) \hat{\phi}_2(\mathbf{r})^* \hat{g}(\mathbf{r}'|\mathbf{r}), \tag{4}$$

where

$$\hat{g}(\mathbf{r}'|\mathbf{r}) = \Delta k^3 a^3 \cos\theta \frac{e^{-j\Delta k|\mathbf{r}'-\mathbf{r}|}}{\Delta k|\mathbf{r}'-\mathbf{r}|}.$$

To define the image of an object we need to represent it through a function. As shown in Eq. (4), the acoustic emission has information of the sphere in both high and low ultrasound frequencies. This information is present in the radiation force function $\hat{y}(\omega_0)$ and the acoustic outflow $\hat{q}(\Delta\omega)$, respectively. We assume that the acoustic emission of an object (or region) is a linear combination of the emission of every infinitesimal volume in the object. The functions $\hat{y}$ and $\hat{q}$ may vary linearly within the object. Hence, we define the object function as

$$\hat{o}(\mathbf{r}; \omega_0, \Delta\omega) = \hat{y}(\mathbf{r}, \omega_0) \hat{q}(\mathbf{r}, \Delta\omega). \tag{5}$$

The vibro-acoustography image of the object is, thus, given by

$$\hat{\imath}(\mathbf{r}; \omega_0, \Delta\omega) = \hat{o}(\mathbf{r}; \omega_0, \Delta\omega) \star \hat{h}(\mathbf{r}), \tag{6}$$

where $\hat{h}(\mathbf{r})$ is the PSF of the system and the symbol $\star$ denotes spatial convolution. It follows from Eq. (6) that the image of an small object brings up its high- and low-frequency characteristics.