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Finite element analysis of left ventricle during cardiac cycles in viscoelasticity

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ABSTRACT

To investigate the effect of myocardial viscoeslasticity on heart function, this paper presents a finite element model based on a hyper-viscoelastic model for the passive myocardium and Hill's three-element model for the active contraction. The hyper-viscoelastic model considers the myocardium microstructure, while the active model is phenomenologically based on the combination of Hill's equation for the steady tetanized contraction and the specific time–length–force property of the myocardial muscle. To validate the finite element model, the end-diastole strains and the end-systole strain predicted by the model are compared with the experimental values in the literature. It is found that the proposed model not only can estimate well the pumping function of the heart, but also predicts the transverse shear strains. The finite element model is also applied to analyze the influence of viscoelasticity on the residual stresses in the myocardium.

1. Introduction

Cardiovascular diseases, being the leading cause of death and morbidity, have been studied from different aspects, such as diagnostic tests, surgical procedures, and medications. Both from clinical practice and laboratory research it has been realized that the mechanical performance of the ventricular muscle is one of the most important factors that can influence the pumping function of the heart. For instance, the weak contractive ability can cause a reduced ventricular ejection fraction, and the high stiffness of the myocardium muscle can cause impaired filling [43]. Thus, to better treat cardiovascular diseases, it is essential to investigate the relation between the heart's function and its mechanical properties. In general, stress and strain are two typical kinds of parameters in the mechanical characterization of local myocardial properties, and have been used as the vital determinants of many cardiac physiological and pathophysiological functions [15].

To avoid the experimental limitations encountered in the direct measurement of wall stresses in an intact ventricle [16], several mathematical models for the passive left ventricle (LV) have been presented to predict the stresses. Earlier mathematical models based on the shell analysis theory paid attention to the overall pressure–volume behavior of the LV. This kind of global model has been explored extensively and used in clinical diagnosis [20]. With the development of continuum mechanics, especially the finite element method, mathematical models quantified in terms of local deformation and stress have been proven to be able to reflect some characteristics of the ventricular mechanics. Up to now, a few local-property models, including the pole–zero model [24], Fung-type models [30], and strain–energy function [4], have been proposed in the finite elasticity framework to describe the heterogeneous, anisotropic properties of the LV. Recently, a structurally based model of the passive myocardium, which takes into account the morphology and structure of the myocardium, was proposed by Holzapfel and Ogden [14]. Due to its simple invariant-based formulation and its small set of material parameters, the Holzapfel–Ogden model is particularly attractive in present practice [28]. In fact, as Fung mentioned, the myocardium is viscoelastic. Although the viscoelastic property of the myocardium is closely related to its physiological functions, especially in situations involving a high heart rate, the relevant study is still immature. To the author's knowledge, there are two viscoelastic myocardial models, both constructed in the quasi-linear viscoelasticity theory, that can be found in the literature [17,23].

To understand the active ventricular mechanics during systole, models characterizing the contractile properties of the cardiac muscle fibers are necessary. At present, several current models of cardiac muscle contraction under different assumptions, e.g., the time-varying elastance model, the modified Hill's model, and the fully history-dependent model, can be found in the literature. The time-varying elastance model describing the relation of the LV pressure to the LV volume and a slack volume reflects a load-

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independent intrinsic contractility property of the LV [27]. The modified Hill's model predicts the fiber stresses by modifying shortening or lengthening of the sarcomere based on the force–velocity relation [4]. The fully history-dependent model proposed by [10] is a general deactivation model of cardiac contraction in rapid length perturbation.

Based on active and passive models, various numerical techniques, such as the finite element (FE) method, have been used to estimate the deformation patterns of the LV under different circumstances. For instance, Vetter and McCulloch [40] developed an anatomically detailed FE model to analyze the stress state during passive filling of the left ventricle. Bovendeerd et al. [2] developed an FE model in which the LV is approximated by a truncated confocal ellipsoid, and studied the influence of the fiber orientation on the LV mechanics. Dorri et al. [5] constructed an FE model for a representative human ventricle to investigate the role played by the LV myocytes in cardiac function. Jiang et al. [19] used the smoothed finite element method to guarantee the accuracy of the analysis of the passive LV to be volumetric locking-free.

As an important functional indicator of various biological organs, residual stresses/strains have been extensively investigated from the aspects of experimental measurement and theoretical analysis. Rachev and Greenwald [29] reviewed the experimental methods for measuring the residual stresses in the artery wall, and found that residual stresses can increase arterial compliance, thereby making it more suitable to be a blood conduit. The residual strains in the myocardium were well illustrated by Omens and Fung [25], who radially cut an equatorial slice of rat LV, and defined the opening angle as a residual strain index. Summerour et al. [37] followed Omens and Fung's line to study how the remodelings of collagen fibers and myocytes change the residual strains in the ischemic ventricular myocardium. Costa et al. [4] developed an experimental procedure to measure the transmural distribution of the residual strains by a biplane radiographic scanning of the coordinates of beads implanted in the heart wall. To incorporate residual stresses into the constitutive modelling, Nash and Hunter [24] introduced a growth tensor that can map the load-free configuration to the stress-free configuration in the analysis of the beating heart. Alternatively, Wang et al. [42] applied the virtual configuration approach proposed by Hoger [12] to an FE analysis based on the Holzapfel–Ogden model.

In this paper, efforts are made to combine an orthotropic finite-viscoelastic model for the passive myocardium developed by the authors and a modified Hill's model for the active contraction to analyze the deformations and stresses experienced by the LV over the whole of the cardiac cycles. Special attention is paid to the residual stresses and the end-diastolic strains, whose shear components are hard to correctly predict by an elastic constitutive model. In addition, to improve the stability and convergence of the FE analysis, the consistent tangential moduli of the modified Hill model are derived and implemented.

2. A hyper-viscoelastic model of the passive myocardium

The viscoelastic model of the passive myocardium used to analyze the LV is briefly described in this section. For its detailed derivation and application to a cylindrical LV model, cf. [31].

2.1. The decoupled free-energy function

From a morphological and microstructural standpoint, Holzapfel and Ogden [14] have pointed out that the passive myocardium tissue can be locally characterized as an orthotropic material in the reference configuration. The orthotropy of the myocardium is also illustrated by the experiments about the

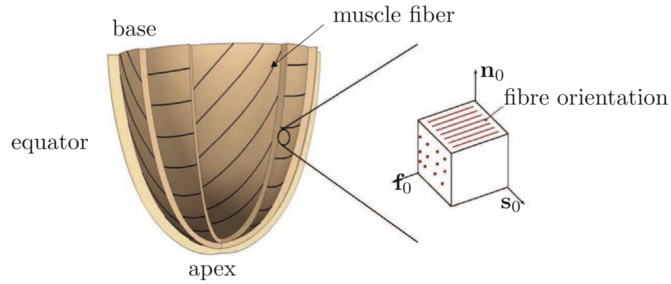

Fig. 1. Schematic view of the fiber distributions across the LV.

myocardial microstructural architecture [22] and the slippage of adjacent muscle layers along the cleavage planes [21]. Therefore, there are three mutually orthogonal preferred material directions: the fibre axis $\mathbf{f}_0$, the sheet axis $\mathbf{s}_0$, and the sheet normal axis $\mathbf{n}_0$, as shown in Fig. 1. It is well known that the isotropic constitutive relation merely depends on the deformation quantities, but the anisotropic relation is simultaneously affected by both the deformation quantities and the intrinsical material structural quantities. In general, the structural quantities can be represented by the structural tensors defined by the tensor products of the preferred direction vectors. In orthotropicity, three irreducible structural tensors, $\mathbf{L}_0$, $\mathbf{M}_0$, and $\mathbf{N}_0$, can be expressed as

$$\mathbf{L}_0 = \mathbf{f}_0 \otimes \mathbf{f}_0, \quad \mathbf{M}_0 = \mathbf{s}_0 \otimes \mathbf{s}_0, \quad \mathbf{N}_0 = \mathbf{n}_0 \otimes \mathbf{n}_0 \tag{1}$$

The symbol $\otimes$ denotes the tensor product of two vectors.

To establish the thermodynamically viscoelastic constitutive relation, the Helmholtz free energy function Ψ, which is the thermodynamical counterpart of the elastic strain energy, has to be suitably postulated for the specific material. In this paper, if we only consider isothermal mechanical phenomena, then orthotropicity leads to $\Psi = \Psi(\mathbf{C}, \mathbf{L}_0, \mathbf{M}_0, \mathbf{N}_0, \mathbf{Q}_1, \ldots, \mathbf{Q}_q)$, which is a scalar-valued function with $4 + q$ tensor arguments. Among these, $\mathbf{C}$ is the right Cauchy–Green strain tensor, and the internal variables $\mathbf{Q}_1, \ldots, \mathbf{Q}_q$, defined in the reference configuration, represent the dissipative mechanism associated with the viscoelasticity.

Based on the experimental evidence that the bulk deformation of soft tissue is elastic, and the viscoelasticity mainly occurs in the deviatoric deformation, the free energy function can be decoupled in the physical sense as [13]

$$\Psi(\mathbf{C}, \mathbf{L}_0, \mathbf{M}_0, \mathbf{N}_0, \mathbf{Q}_1, \ldots, \mathbf{Q}_q) = \Psi^0_{\text{vol}}(J) + \Psi^0_{\text{iso}}(\hat{\mathbf{C}}, \mathbf{L}_0, \mathbf{M}_0, \mathbf{N}_0, \mathbf{Q}_1, \ldots, \mathbf{Q}_q) - \frac{1}{2}\sum_{i=1}^{q} \hat{\mathbf{C}} : \mathbf{Q}_i + \Xi\left(\sum_{i=1}^{q} \mathbf{Q}_i\right) \tag{2}$$

where J is the Jacobian determinant of the deformation gradient, $\hat{\mathbf{C}}$ is the isochoric right Cauchy–Green tensor, Ψ^0_{vol} is the initial free-energy function describing the bulk elastic response, Ψ^0_{iso} describes the deviatoric elastic response, and Ξ is a given function of the internal variables, which can be determined via the Clausius–Plank inequality. The reason for the introduction of the specific viscoelastic form, $-\frac{1}{2}\sum_{i=1}^{n} \hat{\mathbf{C}} : \mathbf{Q}_i$, in the free-energy function is twofold. First, it is a simple matter to find a function Ξ that makes Ψ satisfy the second law of thermodynamics. Second, this kind of free energy function can be viewed as the finite deformation extension of the linear Maxwell viscoelastic model. Through the Coleman–Noll procedure, the second Piola–Kirchhoff stress and the free-energy function have the relation

$$\mathbf{S} = J\frac{\partial \Psi^0_{\text{vol}}}{\partial J}\mathbf{C}^{-1} + J^{-2/3}\mathbb{P} : \left(2\frac{\partial \Psi^0_{\text{iso}}}{\partial \hat{\mathbf{C}}} - \sum_{i=1}^{q} \mathbf{Q}_i\right) \tag{3}$$

with

$$\mathbb{P} = \mathbb{I} - \tfrac{1}{3}\mathbf{C}^{-1} \otimes \mathbf{C}$$