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Analysis

A two-stage econometric method for the estimation of carbon multipliers with rectangular supply and use tables

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ABSTRACT

The Supply-Use Based Econometric (SUBE) approach was proposed to calculate stochastic input–output multipliers from rectangular supply-use tables under the product technology assumption. However, the resulting total use of direct requirements stimulated by final demand (be they carbon emissions, labor, etc.) may differ from the actual total use of direct requirements. To solve this problem, we propose in this paper a two-stage SUBE approach, which takes as prior the initially estimated SUBE multipliers and obtains a posterior set of two-stage SUBE multipliers by constrained least squares minimization. We illustrate the results with an empirical application for carbon emissions in the Portuguese economy in 2005.

1. Introduction

In the modern system of national accounts (SNA, 2008), source input–output (IO) data is reported in the format of Supply and Use Tables (SUT) while IO multipliers are usually calculated from symmetric IO tables through a Leontief inverse (Miller and Blair, 2009). The conversion from SUT to a symmetric IO table involves the choice of a technology assumption for the treatment of secondary products (Kop Jansen and Ten Raa, 1990; Rueda-Cantuche and Ten Raa, 2009; Ten Raa and Rueda-Cantuche, 2003). A popular strategy is the use of the product technology assumption in symmetric product-by-product tables. Under this assumption the commodities produced by different industries are assumed to follow the same production recipe. This assumption has convenient theoretical properties (Kop Jansen and Ten Raa, 1990), although it may lead to negatives (de Mesnard, 2011). When the number of industries differs from the number of commodities the commodity technology assumption does not yield a solution, although an approximation can be found, by using the SUBE approach described below. An alternative is the use of the Moore–Penrose pseudo-inverse, which was used by (Pereira et al., 2011) in the context of IO analysis and by (Heijungs and Suh, 2002) in the context of life cycle analysis (Suh et al., 2010).

The Supply-Use Based Econometric (SUBE) approach (formalized in (Rueda-Cantuche, 2011) and based on previous applications on economic (Ten Raa and Rueda-Cantuche, 2007) and environmental (Rueda-Cantuche and Amores, 2010) contexts allows the implementation of the product technology assumption in rectangular SUTs and the introduction of uncertainty in the calculation of input–output multipliers.

These are the two main advantages of the SUBE approach against other standard IO applications, like (Wachsmann et al., 2009), among others.

Let n_C be the number of commodities (or products) and n_A the number of industries (or activities) in the economy. The SUBE backward Leontief carbon multipliers (Oosterhaven, 1996) are the elements of vector $\mathbf{b}$ of length n_C defined as:

$$\mathbf{r}' = \mathbf{b}'\left(\mathbf{V}' - \mathbf{U}\right) + \epsilon', \qquad (1)$$

where the elements of vector $\mathbf{r}$ of length n_A are the direct carbon emissions in each industry; $\mathbf{V}$ (of size $n_A \times n_C$) and $\mathbf{U}$ (of size $n_C \times n_A$) are, respectively, the make and use matrices; ϵ is a vector of errors; $'$ denotes transposition and vectors are in column format by default. Each element of $\mathbf{b}$ is the total amount of direct and indirect carbon emissions stimulated by one unit of final demand of a given commodity.

The SUBE multipliers are obtained by an ordinary least squares (OLS) regression. As demonstrated by (Rueda-Cantuche and Amores, 2010), for square SUTs, i.e., $n_A = n_C$ then $\epsilon = 0$ and the OLS solution matches the solution for a standard Leontief quantity model under the commodity-technology assumption:

$$\mathbf{b}' = \mathbf{d}'(\mathbf{I} - \mathbf{A})^{-1}, \qquad (2)$$

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where $\mathbf{A} = \mathbf{U}(\mathbf{V}')^{-1}$ and $\mathbf{d}' = \mathbf{r}'(\mathbf{V}')^{-1}$. $\mathbf{A}$ is the matrix of technical coefficients and $\mathbf{d}$ is the vector of direct carbon emissions per unit of industry output. Total direct carbon emissions are $r = \mathbf{r}'\mathbf{e}$, where $\mathbf{e}$ is a vector of ones of the appropriate size.

Let $\mathbf{y}$ stands for a vector of commodity final demand and $\mathbf{q}$ for a vector of total commodity output, so that $\mathbf{q} = \mathbf{V}'\mathbf{e} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}$. It follows that:

$$\mathbf{b}'\mathbf{y} = \mathbf{b}'\left(\mathbf{V}'-\mathbf{U}\right)\mathbf{e} = \mathbf{d}'(\mathbf{I}-\mathbf{A})^{-1}\mathbf{y} = \mathbf{d}'\mathbf{q} = \mathbf{r}'\mathbf{e} = r. \tag{3}$$

That is, the sum of carbon emissions stimulated by final demand equals the sum of direct carbon emissions that take place in all industries, r.

One of the advantages of the SUBE approach is that it allows the estimation of unbiased and consistent values of $\mathbf{b}$ using rectangular SUTs, i.e., $n_C \neq n_A$, and not only square SUTs, $n_C = n_A$, as under the standard commodity technology assumption. However, Eq. (3) does not hold for the OLS estimate of $\mathbf{b}$. Indeed, only if the goodness of fit of the OLS regression is perfect ($\epsilon = 0$) then it will result that $\mathbf{b}'\mathbf{y} = r$. In general ($\epsilon \neq 0$) and therefore $\mathbf{b}'\mathbf{y} \neq r$.

Because the sum of total carbon emissions stimulated by final demand does not match exactly the total carbon emissions of all industries, IO practitioners may find the use of the SUBE approach difficult to justify. This is especially important within the environmental field (Peters, 2008), where the allocation of environmental impacts of production to final demand has a prominent role.

However, we believe that the theoretical advantages of the SUBE approach (the possibility of using rectangular SUTs and of estimating stochastic IO multipliers under a commodity technology assumption) makes it worthwhile to explore modified versions of the SUBE approach so that Eq. (3) can be fulfilled in the case of rectangular tables too, $\mathbf{b}'\mathbf{y} = r$.

With that purpose, the present paper presents a two-stage version of the SUBE approach which guarantees that Eq. (3) holds, while allowing the estimation of Leontief backward multipliers with rectangular SUTS.[1] In the first step a prior configuration of conventional SUBE multipliers is obtained. In the second step a posterior configuration of multipliers is obtained which is as close as possible to the prior while satisfying Eq. (3).

A fundamental concept underlying the present work is that of a supply-use identity of embodied requirements, introduced to IO analysis in the axiomatic definition of an indicator of environmental responsibility (Marques et al., 2012; Rodrigues and Domingos, 2008; Rodrigues et al., 2006; Rodrigues et al., 2010).

The paper proceeds as follows. Section 2 describes the one-stage SUBE approach and introduces the so called two-stage SUBE approach. Section 3 shows an empirical application to the Portuguese economy and Section 4 concludes. An Appendix A presents a comparison between the SUBE approach and that of (Wachsmann et al., 2009).

2. Two-stage Econometric Multipliers

Under the product-technology assumption a carbon multiplier, b_j, is the total amount of carbon emissions which is stimulated by one unit of final demand of product j. The same multiplier b_j is used for the production of commodity j that satisfies such new final demand, independently of the producing industry i. Moreover, the product multiplier is also the same in the case of intermediate inputs, i.e., the same multiplier b_j applies to the use of commodity j independently of the purchasing industry i.

[1] Notice that the proposed method leads to a slightly biased estimates of $\mathbf{b}$ although with the minimum bias so as to fulfill Eq. (3).

In this sense, from the supply side:

$$\sum_{j=1}^{n_C} b_j V_{ij}$$

would stand for the total carbon emissions generated by industry i. On the other hand, from the use side, this amount must be equal to the sum of direct carbon emissions of industry i plus its associated intermediate input carbon emissions. i.e.:

$$\sum_{j=1}^{n_C} b_j U_{ji} + r_i.$$

This way, it is possible to define a supply-use identity of embodied emissions (the product of a multiplier and a monetary flow) of an industry as:

$$\sum_{j=1}^{n_C} b_j U_{ji} + r_i = \sum_{j=1}^{n_C} b_j V_{ij},$$

or, in matrix format:

$$\mathbf{b}'\mathbf{U} + \mathbf{r}' = \mathbf{b}'\mathbf{V}'. \tag{4}$$

In words, for every industry the sum of emissions embodied in intermediate use and direct emissions of a given industry should be identical to the sum of emissions embodied in the products delivered by the same industry.

Eq. (4) may be re-written as $\mathbf{r}' = \mathbf{b}'(\mathbf{V}' - \mathbf{U})$, which is precisely the deterministic part of Eq. (1). As a result, we take Eq. (1) as our starting point to provide the analytical solution to the SUBE multipliers (hereafter, we will denote them as "one-stage" SUBE multipliers in contrast to the so called "two-stage" SUBE multipliers that will be described afterwards). We denote residuals as the deviations of the estimated industry's carbon emissions with respect to their actual values:

$$\epsilon' = \mathbf{r}' - \mathbf{b}'\left(\mathbf{V}' - \mathbf{U}\right). \tag{5}$$

The solution given in the single-stage SUBE approach relaxes the supply-use identities of embodied emissions given by Eq. (4) and determines the vector $\mathbf{b}$ which minimizes the sum of squared errors, Eq. (5). This is an ordinary least squares (OLS) unconstrained optimization problem, whose objective function is:

$$L_1 = \frac{1}{2}\left(\mathbf{r}' - \mathbf{b}'\left(\mathbf{V}' - \mathbf{U}\right)\right)\left(\mathbf{r}' - \mathbf{b}'\left(\mathbf{V}' - \mathbf{U}\right)\right)'.$$

Knowing that the function is concave, the global minimum is obtained by setting all partial derivatives to zero and solving the resulting system for the unknown parameters, b_j. Based on (Johnston, 1984), the OLS estimation of $\mathbf{b}$ is given by:

$$\hat{\mathbf{b}}' = \mathbf{r}'\left(\mathbf{V}' - \mathbf{U}\right)'\left(\left(\mathbf{V}' - \mathbf{U}\right)\left(\mathbf{V}' - \mathbf{U}\right)'\right)^{-1}.$$

It is interesting to observe that the econometric OLS solution described above is exactly the same expression as the Moore–Penrose pseudoinverse used in (Pereira et al., 2011) and (Heijungs and Suh, 2002).

We now look for a way to obtain each b_j (the multiplier of a product j used in intermediate or final consumption), which fulfills the so called global supply-use identity of embodied requirements:

$$\hat{\mathbf{b}}'\mathbf{y} = \mathbf{r}'\mathbf{e}. \tag{6}$$