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Methods

On dimensions of ecological economics

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ABSTRACT

A recent paper (Ecological Economics 69, 2010, pp. 1604–1609) has addressed the issues of dimensional homogeneity of equations and non-linear transformations of variables in economic and ecological economic models. The authors argued that logarithmic transformation cannot be used when variables are dimensional, presented several examples of purportedly incorrect use in applied economics and ecological economics publications, and concluded that these applications "make no sense."

In this paper we show that this view goes against well established theory and practice of many disciplines including physics, statistics, biology, and economics, and rests on an inadequate understanding of dimensional homogeneity and the nature of empirical modeling in applied sciences. We believe that it is important to clarify that the use of dimensional variables in transcendental functions is in fact in accordance with the established scientific consensus so as to prevent further confusion from arising in ecological economics where addressing complex problems requires the synthesis of insights from many diverse disciplines to further our understanding of the environment–economy interface.

We also provide novel applications of dimensional methods to ecological economics and useful methodological references from several strands of scientific literature, not previously systematically consolidated, that should be of interest to every applied researcher.

1. Introduction

To further our understanding of the interdependence between the economy and the environment, ecological economics often seeks mathematical relationships among quantities that describe the phenomena under investigation. These quantities, variables and constants in our models, will often be *dimensional* in nature, i.e., their numerical value depends on the unit of measurement chosen. In a recent paper, published in this journal (Ecological Economics 69, 2010, pp. 1604–1609), Mayumi and Giampietro, hereinafter referred to as M&G, using arguments based on the Taylor's theorem of calculus, argued that exponential and logarithmic functions can only be applied to dimensionless numbers. They then reviewed several economics and ecological economics papers published over the past 50 years where this precept is purportedly not followed, resulting in applications that, according to the authors, "make no sense," and concluded that "it is unfortunate that many empirical and theoretical studies in economics, as well as in ecological economics, use dimensional numbers in exponential or logarithmic functions" and that "economists concerned with the biophysical and monetary aspects of ecological and economic interactions must understand the importance of *dimensional homogeneity*."

These are extraordinary claims that, if correct, would imply that most applications of statistics, economics, econometrics, and a considerable number of application in physics, which routinely employ logarithmic transformations of dimensional variables to model observed phenomena, simplify expressions, gain compliance with common statistical assumptions, estimate model parameters, and test hypotheses against observed data, among other things, are, using the authors' own words, "unacceptable." Relationships that capture the essence of ecological economics such as the stochastic IPAT (see, e.g., Dietz and Rosa, 1994; York et al., 2003) and the environmental Kutznets curve (see, e.g., Grossman and Krueger, 1993; Stern, 2010), where logarithms of dimensional variables are an essential part of the analysis, would also be unacceptable.

In this paper we show that the use of dimensional variables as arguments to transcendental functions in the examples criticized by M&G is in fact in accordance with the established scientific consensus. In the next section we review the concept of homogeneity of equations with physical quantities within traditional dimensional analysis and its extension to social sciences and economics. In Section 3 we apply dimensional analysis to economics and ecological economics problems and show how it can be useful in defining key variables, helping to construct models, and checking the "physical" validity of equations. In Section 4 we look at why often, logarithmic transformations of dimensional quantities that appear to be violate homogeneity, are actually part of a homogeneous expression. We also discuss, within an example from ecological economics, the role of dimensional constants. In

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Section 5 we look at nonhomogeneous models and empirical equations, their usefulness, and their correct interpretation. We show that, because of the complexity of the environment economic interactions, it is a misuse of dimensional analysis to insist that homogeneity rules must be rigidly and uncritically applied. We conclude with Section 6.

As M&G ignored the whole of the vast literature on dimensional analysis, we will try to amend this important omission, by providing useful references to a large body of literature scattered in various disciplines. The standard reference in dimensional analysis remains Bridgman (1931). Useful treatments of the topic include Langhaar (1951), Palacios (1964), de St. Q. Isaacson and de St. Q. Isaacson (1975), and Barenblatt (1996). For a historical account of the method see Macagno (1971) and Roche (1998). In economics, the earliest treatment is Jevons (1888). Several authors of economic books, acknowledging its importance, dedicate a chapter on dimensional analysis. They include Allen (1938), Shone (2002), and Neal and Shone (1976). The most authoritative exposition in the field of economics remains the book by De Jong (1967). Useful papers published on the subject in economics include De Jong and Kumar (1972) and Okishio (1982). Other, more specific references, will be provided in later sections.

2. Dimensional Variables and Their Homogeneous Equations

In order to avoid the mistakes in Mayumi and Giampietro (2010) and to apply dimensional methods to ecological economics correctly, we need to better understand the concept of dimensional homogeneity which has its roots in the fundamental theory of measurement in physics. Central to this understanding is the concept of "physical quantity." Note that in this context this terminology could be the source of some confusion, as in green accounting and ecological economics, "physical" is synonymous with "material" or "embodied energy" usually contrasted with monetary values as in Weisz and Duchin (2006). It is better to think of physical quantity as cardinally measurable property so that it becomes clear that it can include economic quantities such as "goods" and "money."

We need to address the following questions.

- What kind of numerical values representing physical properties can be considered a physical quantity, and
- what kind of restrictions apply to equations between physical quantities.

Most physical quantities have several units of measurement that are routinely employed in applications. We will use the expression *dimensional quantity*, to refer to a quantity whose numerical value depends on a specific unit of measurement. Following a well established convention, we will keep the concepts of dimensions and units distinct. A unit of physical quantities will be a standard for measurement of the same physical quantity, in the usual sense, like a meter or a kilogram. Dimensions can be regarded as generalized units. For example, anything that could be measured in mass units, such as kilograms, is considered to have the dimensions of mass, that we will denote with the symbol $[M]$.[1]

2.1. Base Quantities

Physical quantities are classified into two types: base quantities and derived quantities. The dimensions of basic quantities in classical mechanics are usually length $[L]$, mass $[M]$, and time $[T]$. The base quantities form a complete set of basic components for an open-ended system of "derived" quantities. This triplet LMT can be considered a *system of units* if it is sufficient to express all other quantities of interest in a specific field. It is important to stress that a given system of units is, to some extent arbitrary, and defined by convention. The base quantities are defined in entirely physical terms. Two physical quantities of the same kind, x_0 and x_1, are comparable if the following "ratio" is operationally and uniquely defined,

$$\frac{x_1}{x_0} \Longleftrightarrow \lambda, \tag{1}$$

where λ is a numerical value indicating that x_1 is λ times greater than x_0.[2] This "axiom" is at the basis of the process by which physicists assign numbers to properties of objects. If we take the quantities x_0 to be a "standard unit," we say that the process of measurement produces the numerical value λ, the *number of measurement*. If we change units, say from x_0 to x_0', though the number of measurement will change, the quantity itself remains "physically" unaffected. Also, the ratio of any two samples of a base quantity remains constant when the base unit size is changed. In statistics and social sciences, quantities satisfying this property are said to be measured on a *ratio scale* (see, e.g., Hand, 2004; Stevens, 1946). When fundamental quantities of the same type are physically equated or added together, their corresponding numbers of measurement satisfy equations of the same form,

$$\begin{array}{ccc} x_1 = x_2 & \Longleftrightarrow & \lambda_1 = \lambda_2 \\ \underbrace{x_2 + x_3 = x_4}_{\substack{\textit{Physical operations}\\ \textit{(comparison and}\\ \textit{concatenation) on}\\ \textit{physical quantities}\\ \textit{of the same type}}} & \Longleftrightarrow & \underbrace{\lambda_2 + \lambda_3 = \lambda_4}_{\substack{\textit{Math operations}\\ \textit{(equality and addi-}\\ \textit{tion) on numbers of}\\ \textit{measurement}}} \end{array}$$

with x_i/x_0 giving λ_i, for $i=1,\ldots,4$. Note that if the unit of measure is changed, so that x_i/x_0' produces λ_i', for $i=1,\ldots,4$, the form of the equations remain unchanged: $\lambda_1'=\lambda_2'$ and $\lambda_2'+\lambda_3'=\lambda_4'$. If the above ratio is not defined, equalities might become inequalities by a simple change of units. In that case, the equation will be valid only for the particular choice of units. It is taken for granted that only quantities measured on a ratio scale are amenable to dimensional analysis (see, e.g., Bridgman, 1931; Krantz et al., 1971).

Outside basic physics, the choice of fundamental dimensions to adopt is far less clear and will depend on the area of application. For example, in macroeconomics, time $[T]$, money $[\$]$, goods $[R]$, and utility $[U]$, might be sufficient to derive all other quantities. See De Jong (1967) and Neal and Shone (1976) for a more detailed discussion. To apply dimensional analysis, we choose to treat many economic and social measurements as ratio scales, though they involve substantial pragmatic consideration. Many quantities that appear in ecological economics models are more appropriately measured on other scales, such as, following the well-known classification in Stevens (1946), the nominal, ordinal, and interval scales. As an example, though sums of money can be considered ratio scales, it does not follow that money, say, as a measure of utility in the exchange of goods, is also a ratio scale. In fact, research by Kahneman and Tversky (1979) has shown that zero is not an absolute reference point for monetary measurement, which would make the scale an interval one. There is a long debate in economics on cardinal and ordinal utility (see, e.g., Allen, 1956). Other variables used in ecological economics that are measured on the interval scale such as temperature (degrees Celsius, and Fahrenheit, but not with the Kelvin scale which has an "absolute" zero),[3] ordinal scale, such as "intelligence" in a growth model (as an example, IQ as in Morse, 2006), and on nominal scales, such as the political variables in a

[1] This square bracket notation dates back to Maxwell (see, e.g., Macagno, 1971) and will be used here to denote equivalence class as in Langhaar (1951) and De Jong (1967). Alternatively, it can be interpreted as a function meaning "dimension of."

[2] The division symbol here should be interpreted as a physical operation.

[3] Differences in temperature could be used instead.