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Semi-computational simulation of magneto-hemodynamic flow in a semi-porous channel using optimal homotopy and differential transform methods

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ABSTRACT

In this paper, the semi-numerical techniques known as the optimal homotopy analysis method (HAM) and Differential Transform Method (DTM) are applied to study the magneto-hemodynamic laminar viscous flow of a conducting physiological fluid in a semi-porous channel under a transverse magnetic field. The two-dimensional momentum conservation partial differential equations are reduced to ordinary form incorporating Lorentizian magnetohydrodynamic body force terms. These ordinary differential equations are solved by the homotopy analysis method, the differential transform method and also a numerical method (fourth-order Runge–Kutta quadrature with a shooting method), under physically realistic boundary conditions. The homotopy analysis method contains the auxiliary parameter $\hbar$, which provides us with a simple way to adjust and control the convergence region of solution series. The differential transform method (DTM) does not require an auxiliary parameter and is employed to compute an approximation to the solution of the system of nonlinear differential equations governing the problem. The influence of Hartmann number (*Ha*) and transpiration Reynolds number (mass transfer parameter, *Re*) on the velocity profiles in the channel are studied in detail. Interesting fluid dynamic characteristics are revealed and addressed. The HAM and DTM solutions are shown to both correlate well with numerical quadrature solutions, testifying to the accuracy of both HAM and DTM in nonlinear magneto-hemodynamics problems. Both these semi-numerical techniques hold excellent potential in modeling nonlinear viscous flows in biological systems.

1. Introduction

Magneto-hemodynamics involves the flow of electrically-conducting physiological fluids under the action of applied magnetic fields [1]. The conducting nature of blood has been confirmed to be generated by the presence of iron in the hameoglobin molecule as described in [2–4] and also ionic content. Many researchers have focused on exploiting this phenomenon to control blood flow velocities in surgical procedures, magnetic drug targeting [5] and evaluating the influence of strong magnetic fields on the human circulation system. It has also been established that astronauts experience a decrease in heart rate during space-craft re-entry owing to exposure of the body to the strong magnetic fields in the planetary ionosphere. A number of analytical and computational studies of magneto-hemodynamic flows have been communicated. The historical development of this subject was first presented by Bég [6]. It has also been described more recently in Bég et al. [7] with particular emphasis on cross-diffusion effects in magneto-physiological boundary layers. Sigman et al. [8] were among the first clinical researchers to identify the strong electrically-conducting nature of flowing blood. More recently magneto-hemodynamics has been investigated by Visser [9] and Yamamoto et al. [10]. With rapid modern developments in micro-electro-mechanical systems (MEMS) and nano-technology, there has been a strong resurgence in magnetic flow devices in medical engineering. These devices utilize the Lorentzian body forces associated with magnetohydrodynamics (MHD). Recent examples of MHD biomedical applications include peristaltic micro-magnetic pumps [11] and magnetic pharmacological targeting [12]. Newtonian viscous hydromagnetic physiological flow studies have been communicated by Ramamurthy and Shanker [13], Takhar et al. [14] who employed ferrohydrodynamic (biomagnetic) theory, Bhargava et al. [15] who studied pulsatile heat transfer in magnetic blood flow and Sud et al. [16] for elastic arteries. Tripathi and Bég [17] numerically the transport of conducting blood in a peristaltic pump processing system. Similar

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Nomenclature

DTM	differential transform method
HAM	homotopy analysis method
NUM	numerical method
$\mathscr{L}$	linear operator of the HAM
N	nonlinear operator of the HAM
P	pressure
q	mass transfer parameter (wall transpiration velocity)
Re	Reynolds number
U	dimensionless velocity in the x direction
V	dimensionless velocity in the y direction
h	suspension height
Ha	Hartman number
L_x	length of the slider
u_0	x velocity of the pad
u	dimensionless x-component velocity
v	dimensionless y-component velocity
u^*	velocity component in the x direction
v^*	velocity component in the y direction
x	dimensionless horizontal coordinate
y	dimensionless vertical coordinate
x^*	distance in the x direction parallel to the plates
y^*	distance in the y direction parallel to the plates

Greek symbols

ρ	biofluid density
ν	kinematic viscosity of biofluid
σ	electrical conductivity of biofluid
ε	aspect ratio h/L_x

studies were reported by Mekheimer [18]. Hoque et al. [19] analyzed the magnetohydrodynamic curved duct flow problem using spectral methods, motivated by applications in bioastronautics. Kainz et al. [20] developed a numerical solver to calculate the magneto-hydrodynamic (MHD) signal produced by moving conductive blood flow in the great vessels of the heart, under a static magnetic field. Das and Saha [21] studied magnetohydrodynamic pulsating blood flow in a rough thin-walled distensible conduit.

Viscous *non-conducting* hemodynamic flow in porous tubes or channels has also received much attention in recent years owing to applications in dialysis of blood in artificial kidneys [22], flow of blood in the capillaries [24] and blood transport in oxygenators [23]. Berman [25] presented a classical study in which he obtained an exact solution of the Navier–Stokes equation for steady two-dimensional laminar flow of a viscous, incompressible fluid in a channel with parallel, rigid, porous walls driven by uniform, steady suction (or injection) at the walls. More recently, Desseaux [26] extended Berman's study to analyze the influence of a magnetic field on laminar viscous flow in a semi-porous channel.

The above hemodynamic simulations have generally modeled flow phenomena with systems of ordinary or partial differential equations. In the present study we employ Berman's similarity transformation and study *Berman–Desseaux* flow i.e. *magneto-hemodynamic flow in a semi-porous channel*. This allows the reduction of the governing partial differential equations into a set of *coupled ordinary nonlinear differential equations*. In most cases, such problems will not admit analytical solution, and this necessitates the implementation of special techniques. In recent years, much attention has been devoted to the newly developed methods to construct an analytical (i.e. semi-numerical) solution of differential equation systems in viscous flow. Liao [27] introduced the basic ideas of the homotopy in topology to propose a general analytic method for nonlinear problems, namely the *homotopy analysis method* (HAM) which avoids the need for a “small” parameter. This method has been successfully applied to solve many types of nonlinear problems in biomedical fluid deformable artery flow [28], peristaltic rheological transport [29], endoscope hydrodynamics [30], viscoelastic tribology [31] and magneto-tribology [32]. HAM has also been modified and simplified versions (i.e. homotopy perturbation method (HPM)) employed to resolve successfully many other complex problems in biophysics. Siginificant communications in this regard include Rathore et al. [33] who applied the Sumudu transform modified HAM to Fokker–Planck equations. Singh et al. [34] considered gas flows, Singh et al. [35] used HPM for the fractional reaction-diffusion equations, Singh et al. [36] who studied linear and nonlinear Klein–Gordon equations with HPM and Singh et al. [37] who analyzed the fractional Fornberg–Whitham equation. Other important studies employing semi-numerical methods and spanning a tremendous range of equation systems relevant to medical physics and theoretical biology include Kumar et al. [38], Kumar et al. [39], Singh et al. [40], Kumar et al. [41], Kumar and Singh [42] and Singh et al. [43]. Tripathi et al. [44] have also recently used homotopy analysis and benchmarked with other semi-numerical methods for peristaltic rheological gastric flow simulation.

Another powerful *semi-numerical* method, the differential transform method (DTM), which is based on Taylor series expansions, was developed by Zhou [45] originally in the context of linear and nonlinear problems in electrical engineering. DTM gives exact values of the nth derivative of an analytical function at a point in terms of known and unknown boundary conditions in a fast manner. The differential transform is an *iterative procedure* for obtaining analytic Taylor series solutions of differential equations. Chen and Ho [46] developed this method for partial differential equations and Ayaz [47] applied it to nonlinear systems of ordinary differential equations. Jang et al. [48] presented two-dimensional DTM solutions of partial differential equations. Erfani et al. [49] studied off-center Von Karman swirling stagnation flow using DTM. DTM has also successfully resolved a variety of biological flow problems. Rashidi et al. [50] investigated biomagnetic micropolar blood flow in a haemtological separation (filtration) device using DTM and Padé approximants. Other interesting studies employing DTM simulation include Rashidi et al. [51] who considered biochemical reactors, Rashidi et al. [52] who studied swirling magnetic propulsion flows in biofuel hybrid reactors, and Rashidi et al. [53] who analyzed nanoscale drug biopolymer flow manufacturing systems. Bég et al. [54] who further presented one of the first studies of multi-phase hemodynamic separation processes using DTM.

In the present article, we review briefly the fundamentals of HAM and the DTM and then apply both semi-computational methods to solve the transformed coupled ordinary nonlinear differential equations for laminar magneto-hemodynamic, viscous flow in a semi-porous channel. Both techniques are shown to achieve a rapidly convergent series and the special attributes of each method are elaborated. The present study aims to expose researchers to the exceptional potential of both methods in modern magneto-hemodynamic modeling.

2. Mathematical formulation

Consider the laminar two-dimensional stationary flow of an electrically-conducting incompressible viscous fluid in a semi-porous