

# Adaptive Expectations and Dynamic Models for Network Goods

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*Abstract:* In this work, we consider a utility function that is influenced by the value of network externality function at the consumer's expected market size/share. Using this utility function, a market share adjustment function is introduced and is utilized to develop dynamic models of existing rational and static expectation processes. In addition, the role and scope of dynamic models of market share adjustment process are extended to the well-known adaptive expectation and its extension processes. The properties of equilibrium states of dynamic models are investigated which include location, stability, oscillation and the initial states in systematic and unified way. The most significant byproduct of presented results is that the properties of equilibrium states depend on the type of consumer expectation of a network good and the parameters of dynamic market share adjustment processes.

## I. INTRODUCTION

With regard to network goods, the utility function is known to be affected by the market size/share (Shy 2001). The fundamental concept of network externality is that consumers gain more benefit when more consumers consume the same good. Hence, the market size/share is a factor that affects the consumer's decision to adopt a network good. In practice, consumers do not have information about the actual market size/share. In view of this, consumers need to speculate their expected market size/share.

It is well-known (Katz and Shapiro 1985) that in the network goods economy, the idea of dependence of consumer surplus function on the value of network externality function at the consumer's expected market size/share was introduced. Moreover, under the fulfilled expectation assumption, the rational equilibrium is characterized for compatible products. Katz and Shapiro (1985) ideas and results are extended by Easley and Kleinberg (2010) to consumer demand goods and Amir and Lazzati (2011) to industry performance. Furthermore, under the rational expectation assumption, the effects of the market structure of the equilibrium states are analyzed.

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In this work, we extend the idea of Katz and Shapiro (1985) to modify the utility function that depends on a value of network externality function at an expected market size/share at a real time  $t$ . The consumer's expected market size/share is speculated, and it is based on either the current or the past market share, or the combination of the current and past market share. Instead of directly using properties of modified utility function, we utilize the modified function to introduce a concept of market share adjustment function. Employing the market share adjustment function, we develop mathematical models corresponding to consumer rational, static, and current and lagged adaptive expectations in a systematic and unified way. Furthermore, we establish the existence and location of equilibrium states and study the qualitative properties of equilibrium states in a systematic way. The qualitative properties (stability and oscillation) are analyzed in the context of the parameters of dynamic systems. In fact, the parameters of the dynamic systems are decomposed into subsets that are characterized by the stability and oscillatory regions. Moreover, the presented dynamic model formation provides a suitable design to develop an agent-based simulation model (Paothong 2013).

In short, the paper is organized as follows. By introducing a market share adjustment function, dynamic models for rational and static expectation processes are developed in Section II. In addition, sufficient conditions are given to exhibit the existence of corresponding rational and static equilibrium states. In Section III, by considering a market share adjustment function in the context of adaptive expectations, dynamic models of current and lagged adaptive expectation processes are developed. Furthermore, it is shown the static equilibrium states are invariant under the current and lagged adaptive expectation processes. In Section IV, the sufficient conditions are given to establish the qualitative properties (stability and oscillatory) of dynamic systems. We note that the stability conditions are in terms of parameters of dynamic systems. In Section V, by extending the domain of the speed of adjustment parameter, we develop the results parallel to the results of Section IV. Moreover, we decompose the domain of dynamic systems and the speed of adjustment parameters into regions according to qualitative properties of dynamic systems. In Section VI, by using simulations, we exhibit the influence of the initial states on the solution paths of dynamic models under static, current and lagged adaptive expectation processes.

## II. MODEL

We analyze an underlying network good model in discrete time,  $t \in \{0, 1, 2, \dots\}$ , and all variables are time varying or time invariant. For each  $t$ , a utility function is composed of the consumer's individual preference, the price of the network good and the value of network externality function at a consumer's expected market size/share. In this paper, for simplicity, we treat the market size as the market share,  $n_t$ , that is, it has value between zero and one,  $n_t \in [0, 1]$ . Hence,

$$u(v, n_t^e) = v + g(n_t^e) - p, \quad (2.1)$$

where  $v$  is the consumer's individual preference;  $n_t^e$  is the consumer's expected market share at time  $t$ ;  $p$  is the price of the network good and  $g$  is the generalized network externality function (GNEF) (Paothong and Ladde, 2012). We further make two assumptions:

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