Indeterminacy, capital maintenance expenditures and the business cycle

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ABSTRACT

This paper examines how maintenance expenditures affect the occurrence of indeterminacy in a two-sector model economy, motivated by the empirical fact that equipment and structures are maintained and repaired. McGrattan and Schmitz's (1999) survey on 'Capital and Repair Expenditures' in Canada indicates that maintenance expenditures account for a substantial fraction of output and new investment. It is shown that the endogenous maintenance expenditures reduce the requirement of the degree of increasing returns to scale to generate sunspot equilibria. In fact, the minimum level of the returns to scale required could be as low as 1.0179. This aspect is important since empirical works such as Basu and Fernald (1997) suggest that returns to scale is close to constant.

1. Introduction

Recent years have witnessed the formulation of business cycle models with multiple equilibria. In particular, many researchers explore the mechanisms that give rise to indeterminacy.[1] It has been recognized that the indeterminacy could arise if the assumption of a perfect market is relaxed. In earlier research such as Benhabib and Farmer (1994), the existence of a continuum of equilibria relies on a high degree of increasing returns to scale in production. However, empirical work by Basu and Fernald (1997) depicts that the presence of production externalities is rather modest, if any, which led researchers to pursue model structures with lower scale economies to induce indeterminacy. The increasing returns to scale are often exhibited via external effects.

This paper works on such a model. It examines how maintenance expenditures affect the occurrence of indeterminacy in a two-sector model economy. This model provides an extension of the two-sector, endogenous capital utilization model of Guo and Harrison (2001). The main feature of this model is that the capital depreciation rate varies with capital utilization rate and maintenance expenditures, whereas in many other two-sector model papers the evolution of the depreciation rate is solely determined by variable capital utilization. McGrattan and Schmitz (1999) define maintenance expenditures as "the expenditures made for the purpose of keeping the stock of fixed assets or productive capacity in good working order during the life originally intended". Licandro and Puch (2000) point out that such expenditures are important factors affecting depreciation, as machines are better pre-

served if maintenance activity is engaged during the production process. In the model, the amount of maintenance expenditures affects the capital accumulation law and is upon the representative agent's optimal decisions.

Empirical studies affirm the importance of maintenance expenditures. McGrattan and Schmitz (1999) conduct a survey on 'Capital and Repair Expenditures' in Canada and show that expenditures on maintenance activity are large relative to that on other activities. In this survey, total maintenance and repair expenditures accounted for 5.7 percent of gross domestic product (GDP) over 1981–1993.[2] Over the same period, these expenditures averaged about 28 percent of spending on new investment. Expenditures on R & D were 1.4 percent of GDP which was much lower than maintenance-to-GDP ratio. Moreover, the proportion of public spending on education was 6.8 percent which was only slightly higher than that of maintenance expenditures, indicating that maintenance expenditures are 'too big to ignore'.

This model relates to Guo and Lansing (2007). They investigate the indeterminacy properties of a one-sector model with maintenance expenditures. As there is a lack of data on maintenance expenditures in the U.S., they calibrate maintenance-to-GDP ratio using Canadian data as the proxy for U.S. data. In this paper I consider a two-sector case as subsequent research has indicated that models with two-sector or multi-sectors of production require much lower increasing returns to obtain indeterminacy.[3] Furthermore, I allow households to make decisions on capital maintenance expenditure as households own the capital, whereas in Guo and Lansing's (2007) economy the sequence of

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[1] Examples include Benhabib and Farmer (1996), Wen (1998), Guo and Harrison (2001).
[2] It accounted for 6.1 percent of GDP if the period covers from 1961 to 1993.
[3] For example, Benhabib and Farmer (1996), and Weder (2000).

maintenance expenditures is the firms' choice. The study has quantitatively shown that maintenance expenditures could reduce the minimum required level of increasing returns to scale. The minimum level of returns to scale is 1.0179 which is close to constant.

It has been criticized that a model combining both two production sectors and variable capital utilization tends to generate an extremely narrow range of increasing returns that give rise to indeterminacy (Guo and Lansing, 2007). Under this circumstance it is not possible to generate pro-cyclical consumption with such low degree of externalities. Therefore, this paper also considers a model variant in which capital utilization is assumed to be constant over time. This model is in fact an extension of Benhabib and Farmer's (1996) model by incorporating maintenance activities into their model specification. The results that indeterminacy requires lower returns to scale in models with maintenance activities than non-maintenance economic variants are robust. Under this formation, the countercyclical consumption puzzle is solved and most features of the model moments are comparable to the U.S. data.

The role of maintenance expenditures on the occurrence of indeterminacy is obvious. Starting from an equilibrium path where the rate of discount equals the overall (net) rate of return on capital. Suppose an optimistic agent believes that there will be an increase in the rate of return on capital, the agent will reallocate resources from consumption to investment. In order to validate the agent's expectations as a new equilibrium, the return on capital has to be actually increased at higher level of economic activity and the associated first order conditions still hold. The model has two major features that could achieve this. Firstly, a mild degree of returns to scale exhibits in the present model economy. The marginal product of capital increases when labour flows from the consumption sector into the investment sector. Secondly, engaging in maintenance activities makes capital better preserved and thus increases its productivity. This results in a higher rate of return on capital as well. Combining these two features, the return on capital can easily increase with higher level of capital stock even if the degree of externalities is small.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 analyzes the local dynamics and the indeterminacy properties. In Section 4 I show the business cycle properties generated from the models and address the cyclicality of consumption issue. Section 5 concludes.

2. The model

The model incorporates maintenance expenditures into Guo and Harrison's (2001) two-sector model. The economy consists of a continuum of identical households who make decisions about consumption, labour hours worked, utilization rate of capital and maintenance expenditures. Households own the capital and lend capital and labour services to firms, taking rent and real wage rate as given. Firms produce consumption and investment goods which are sold to households. Households own the firms and therefore the profits are remitted to households.

2.1. Preferences and household's choices

A representative household chooses the sequences of consumption C_t and hours worked L_t to maximize his lifetime utility

$$\int_0^\infty \left(\ln C_t - \frac{L_t^{1+\chi}}{1+\chi} \right) e^{-\rho t} dt, \tag{1}$$

where χ captures the inverse elasticity of labour supply and ρ is the discount rate. The budget constraint faced by the household is

$$C_t + P_t I_t = r_t u_t K_t + w_t L_t, \tag{2}$$

where I_t is the household's investment in new capital, P_t is the relative price of investment goods in units of consumption goods. r_t and w_t are the rental rate of capital and the real wage rate, respectively. u_t is the rate of capital utilization. Let K_t denotes economy-wide capital stock. The law of motion for capital accumulation is given by

$$\dot{K}_t = I_t - \delta_t K_t - M_t, \tag{3}$$

where M_t is goods expenditure on maintenance. $\delta_t \in (0, 1)$ is the rate of capital depreciation which is variable over time. Following Guo and Lansing (2007), δ_t has the form of

$$\delta_t = \tau \frac{u_t^\theta}{(M_t/K_t)^\phi}, \tag{4}$$

where $\tau > 0$, $\theta > 1$, and $\phi \geq 0$. θ is the elasticity of depreciation with respect to capital utilization. ϕ captures the elasticity of depreciation rate with respect to maintenance cost rate:

$$\phi = -\frac{\partial \delta_t}{\partial (M_t/K_t)} \times \frac{(M_t/K_t)}{\delta_t}. \tag{5}$$

Licandro and Puch (2000) define M_t/K_t as 'the maintenance cost rate' that captures the intensity of maintenance activities. Above form of the depreciation rate implies that the depreciation rate depends on both capital utilization and maintenance activities. Higher capital utilization rate accelerates the depreciation whereas higher maintenance expenditures has the opposite effect (Guo and Lansing, 2007).

Let Λ_t be the co-state variable associated with the Hamiltonian set-up of the household's optimization problem. It is often explained as the shadow price of capital, meaning the marginal utility gain if agent's capital constraint is relaxed. The Hamiltonian set-up is

$$H_t = \left(\ln C_t - \frac{L_t^{1+\chi}}{1+x} \right) + \Lambda_t \left[(r_t u_t K_t + w_t L_t - C_t) P_t^{-1} - \tau u_t^\theta k_t^{\phi+1} M_t^{-\phi} - M_t \right] \tag{6}$$

Then the first-order conditions are given by

$$\frac{1}{C_t} = \Lambda_t P_t^{-1}, \tag{7}$$

$$\Lambda_t w_t P_t^{-1} = L_t^\chi, \tag{8}$$

$$r_t P_t^{-1} u_t = \theta \delta_t, \tag{9}$$

$$\phi \delta_t \frac{K_t}{M_t} = 1, \tag{10}$$

$$\frac{\dot{\Lambda}_t}{\Lambda_t} = \rho - r_t u_t P_t^{-1} + (\phi + 1)\delta_t. \tag{11}$$

The transversality condition is $\lim_{t\to\infty} e^{-\rho t} \Lambda_t K_t = 0$. Eqs. (7) and (8) show the intratemporal trade-off between consumption and leisure. Eq. (9) shows that the household utilizes capital by equating the marginal gain and marginal loss of a change in utilization rate. Eq. (10) indicates that the household equates one unit of good expenditure on maintenance to marginal maintenance cost rate with respect to the depreciation rate. Eq. (11) is the intertemporal Euler equation.

2.2. Production technology

The production functions for the consumption sector and investment sector are given by

$$Y_{ct} = A_t (u_t K_{ct})^\alpha L_{ct}^{1-\alpha}, \tag{12}$$

where $A_t = [(\bar{u}_t \overline{K}_{ct})^\alpha \overline{L}_{ct}^{1-\alpha}]^\eta$ and

$$Y_{xt} = B_t (u_t K_{xt})^\alpha L_{xt}^{1-\alpha}, \tag{13}$$

where $B_t = [(\bar{u}_t \overline{K}_{xt})^\alpha \overline{L}_{xt}^{1-\alpha}]^\eta$.