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Robust minimum variance portfolio optimization modelling under scenario uncertainty☆

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ABSTRACT

Our purpose in this article is to develop a robust optimization model which minimizes portfolio variance for a finite set of covariance matrices scenarios. The proposed approach aims at the proper selection of portfolios, in a way that for every covariance matrix estimate included in the analysis, the calculated portfolio variance remains as close to the corresponding individual minimum value, as possible. To accomplish this, we formulate a mixed-integer non-linear program with quadratic constraints. With respect to practical underlying concerns, investment policy constraints regarding the portfolio structure are also taken into consideration. The validity of the proposed approach is verified through extensive out-of-sample empirical testing in the EuroStoxx 50, the S&P 100, the S&P 500, as well as a well-diversified investment universe of ETFs. We report consistent generation of stable out-of-sample returns, which are in most cases superior to those of the worst-case scenario. Moreover, we provide strong evidence that the proposed robust model assists in selective asset picking and systematic avoidance of excessive losses.

1. Introduction

In the last 20 years, the research activity in robust portfolio optimization is immense (Ghahtarani and Najafi, 2013; Mansini et al. 2014; Ayub et al. 2015; Gorissen et al. 2015). Kolm et al. (2014) reviewed the 60-year course of portfolio optimization and confirmed the persistent portfolio robustness trend that has emerged. Results of Google Scholar queries provide some interesting figures that highlight this current thriving momentum: When searching for *modern portfolio theory*, we obtained 404,000 results, when searching for *portfolio optimization*, we obtained 241,000 results and when searching for *robust portfolio optimization*, we obtained 48,700 results. Hence, we note that there is a constantly growing underlying research momentum in the field of robust portfolio optimization.

Recent developments in the field of robust portfolio theory imply that the knowledge of future returns and variances, delivered by classic point-estimation techniques, cannot be thoroughly trusted. Since risk and return are characterized by randomness, one should keep in mind that problem data could be described by a set of scenarios. Mulvey et al. (1995) were the first to work on models of mathematical optimization where data values come in sets of scenarios, while explaining the concept of robust solutions and introducing the robust model formulation.

Tütüncü and Koenig (2004) described asset's risk and return using continuous uncertainty sets and developed a robust asset allocation program solved by a saddle-point algorithm. Also, Pinar and Tütüncü (2005) introduced the concept of robust profit opportunity in single-period and multi-period formulations. Likewise, multi-period portfolio optimization formulations with additional transactional constraints are found in Bertsimas and Pachamanova (2008). Other recent critical works in the field of robust portfolio optimization are those of DeMiguel and Nogales (2009), Rustem and Howe (2009) and Qiu et al. (2015).

While robust optimization is intended to protect the portfolio against uncertainty, Gregory et al. (2011) calculated that it comes with costs in terms of return. In terms of risk, Huo et al. (2012) proposed robust covariance measures to be included in the portfolio optimization process, so as to generate covariance estimates stable and insensitive to outliers. In order to deal with output fluctuations and stress testing with respect to uncertainty in input data, a study of robustness of optimal portfolios under stochastic dominance constraints was con-

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ducted by Dupacova and Kopa (2014). Moreover, Maillet et al. (2015) performed a worst-case minimum variance optimization with respect to alternative covariance matrix estimators.

Kim et al. (2013a) investigated robust models and fundamental factors in order to determine whether robust equity portfolios are more or less sensitive to factors than to individual assets' movements. Moving a step forward, Kim et al. (2014b) proposed robust modeling that allows the control of the level of exposure portfolios have in a factor. Moreover, in a study of composition of robust equity portfolios Kim et al. (2013b) inspected the properties of the selected assets. Kim et al. (2014a) also surveyed developments of robust worst-case optimization, including robust counterparts for value-at-risk and conditional value-at-risk problems. Kim et al. (2015) discussed robust optimization performance with focus on worst market state returns. Another robust worst-case approach within the best value-at-risk Sharpe ratio context is found in Deng et al. (2013).

A very comprehensive review of the 20-year old history of robust portfolio optimization is included in Kolm et al. (2014). Other research articles that summarize recent history and future trends of robust portfolio optimization are those of Fabozzi et al. (2007, 2010) and Scutellà and Recchia (2013), where the relation between robustness and convex risk measures is also studied. A thorough inspection of both theoretical and practical research in robust optimization was made by Ben-Tal et al. (2009).

Besides historical and theoretical reviews, useful guides for practitioners can be also found in Gorissen et al. (2015). In the robust multiobjective field, an effort to characterize the location of the robust Pareto frontier with respect to the corresponding original Pareto frontier using standard multiobjective optimization techniques was made by Fliege and Werner (2014). Finally, we also report other research attempts in the field of robust portfolio optimization, including those of Loulou and Kanudia (1999), Mausser and Laguna (1999), Lobo (2000), Khodadadi et al. (2006) and Ehrgott et al. (2014).

The main goal of this article is to develop a robust minimum variance optimization model that takes into account alternative scenarios of assets' covariances. We focus our study on minimizing portfolio variance under a variety of scenarios, aiming to select assets in a way that portfolio variance remains low, no matter which scenario currently depicts assets' risk features more clearly. For this purpose, we solve a mixed-integer non-linear program, formulated as a quadratic constrained quadratic programming model. We set the proposed program to minimize variance over a number of different time intervals of historical data. In order to make this program more appealing to practical investors, we also use binary decision variables for modeling constraints of cardinality.

Our intention is to examine the effects of this robust portfolio selection procedure in the portfolio construction outcome and to measure its return performance. We backtest our method with abundant historical data of popular indices, such as the EuroStoxx 50, the S&P 100, the S&P 500 and selected combinations of them. Moreover, we apply and backtest on an actively traded mutual fund-of-ETFs, consisting of a total of 150 ETFs. Evidence from the extensive empirical testing implies that the proposed robust methodological framework leads to selective asset picking and conservative out-of-sample returns. The observed returns avoid large inconsistencies and mostly appear to be superior to the worst-case performing scenarios' returns.

The paper proceeds as follows: In Section 2 we present the proposed methodological framework and in Section 3 we proceed with the underlying out-of-sample empirical testing procedure. Finally, a summary of concluding remarks is given in Section 4.

2. Proposed model

2.1. The underlying robust framework

The general framework for robust optimization under scenario uncertainty can be found in Cornuéjols and Tütüncü (2006). Suppose there is a discrete set S of N scenarios, i.e. $S = \{s_1, s_2,\ldots, s_N\}$. The objective function f and the constraint function G depend upon each scenario, while the constraint function is limited within a set K. If the problem was to be solved for a certain scenario $s \in S$, we would have the following problem:

$$\begin{aligned} &\min_x f(x, s) \text{ s.t.} \\ &G(x, s) \in K \end{aligned} \tag{1}$$

The robust formulation of the above problem is given below:

$$\begin{aligned} &\min_{x,t} t \text{ s.t.} \\ &t - f(x, s) \geq 0 \\ &G(x, s) \in K \\ &\forall\ s \in S \end{aligned} \tag{2}$$

In problem (2) the objective functions for all $s \in S$ are suppressed towards zero by use of the variable t. Therefore, a robust choice of the decision variable x is generated. The classical minimum variance problem with a known covariance matrix C is:

$$\begin{aligned} &\min_x x^T C\ x \text{ s.t.} \\ &e^T x = 1 \\ &0 \leq x \leq 1 \end{aligned} \tag{3}$$

where x is the weight vector and e is the column unit vector.

Our purpose is to transform the classical minimum variance problem (3) into formulation (1) and eventually solve the robust counterpart, as expressed in formulation (2).

2.2. Robust minimum variance optimization under scenario uncertainty

We propose the following robust methodological framework; suppose there is a set of scenarios S that describes the assets' performance in the assets' universe. Each scenario $s \in S$ has an expected return vector μ_s and a covariance matrix C_s. We denote σ_s^{*2} as the minimum variance of the portfolio scenario s. This value is obtained by finding the optimal solution of the classical problem for $C = C_s$. The general formulation of the proposed robust program is:

$$\begin{aligned} &\min_{x,t} t \text{ s.t.} \\ &x^T C_s x \leq (1 + t)\sigma_s^{*2} \\ &e^T x = 1 \\ &0 \leq x \leq 1 \\ &t \geq 0 \\ &\forall\ s \in S \end{aligned} \tag{4}$$

The above model provides a solution x which minimizes the portfolio variance, under all scenarios. The variable t indicates the relative worst variance aggravation in the robust choice of weights, i.e. the portion of variance we exchange for robustness. We will refer to variable t as *variance sacrifice*.

2.3. Policy constraints

The fundamental portfolio constraints we have already added are:

The completeness constraint

$$e^T x = 1 \text{ or } \sum_{i=1}^{n} x_i = 1 \tag{5}$$

This is a mandatory constraint, since we assume that all capital needs to be allocated among the n available assets.

The no short sales constraint

$$0 \leq x_i \leq 1,\ \ \forall\ i = 1, 2,\ldots, n \tag{6}$$

This is the usual constraint that prohibits the short selling of assets, i.e.