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A conditional autoregressive range model with gamma distribution for financial volatility modelling[☆]

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ABSTRACT

The commonly used conditional autoregressive range model with Weibull distribution (henceforth WCARR) suffers from serious inlier problem. We conjecture that this problem is due to a misspecified distribution to the disturbance, and propose a conditional autoregressive range model with gamma distribution (henceforth GCARR) to model the volatility of financial assets. In this paper, we first discuss the theoretical properties of the GCARR model and then compare its empirical performance with the WCARR. Empirical studies are performed on a broad set of stock indices in different countries over different time horizons. Consistent with the conjecture, we find that the GCARR model can reduce not only the inlier problem but also the outlier problem of the WCARR model. The results indicate that our GCARR model describes the dynamics of the range-based volatility better than the WCARR model and thus serves as a better benchmark.

1. Introduction

As a measure of risk, volatility plays an important role in both asset pricing and risk management, and has been a central theme in the literature of both financial economics and econometrics. Both academic researchers and policy makers and regulators pay great attention to volatility. To researchers, volatility is the key point to understanding the pricing efficiency of financial markets (Campbell and Shiller, 1988; Cuthbertson and Hyde, 2002). To policy makers and regulators, volatility is closely related to the functioning and the stability of financial markets, which has huge impact on the functioning and fluctuation of the real economy.

Traditionally, volatility estimator is constructed from closing price, and a variety of well-known volatility models have been proposed to describe its dynamics in the last three decades. One of the most phenomenal developments on volatility is the ARCH/GARCH family of models; see Engle (1982); Bollerslev (1986), and Nelson (1991). Bollerslev et al. (1992) make a critical review with thorough survey of ARCH literature. A competitive volatility model to the ARCH is the Stochastic Volatility model (henceforth SV) of Taylor (1986) and Heston (1993). Tsay (2001) discusses the two branches of the literature. Fleming et al. (2001) find that volatility timing strategies outperform the unconditionally efficient static portfolios. For insightful application of volatility models to derivative pricing, see Duan (1995, 1997), Ritchken and Trevor (1999), Heston and Nandi (2000), and Adesi et al. (2008).

Recent academic literature shows a rising interest in using price range to estimate volatility. The idea of using price range in finance can be found in Mandelbrot (1971) who employs it to test the existence of long-term dependence in asset prices. The noticeable application of range to the estimation of volatility can date back to 1980s. Some studies have noted that the price range data can offer a sharper estimate of volatility than the return data. By employing the extreme value theory and some well-known properties of range, Parkinson (1980) forcefully argues and demonstrates the superiority of using range as a volatility estimator as compared with standard methods. Alizadeh et al. (2002) show theoretically, numerically, and empirically that range-based volatility proxies are not only highly efficient, but also approximately Gaussian and robust to microstructure noise. Degiannakis and Livada (2013) find the price range volatility estimator is more accurate than the realized volatility estimator based on five, or less, equidistance points in time[1]. Chou and Liu (2010) empirically investigate the economic value of volatility timing using a range-based volatility model, and find that the range-based volatility model per-

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[1] Other references concerning price range volatility include Gammam and Klass (1980), Wiggins (1991), Rogers and Satchell (1991), Kunitomo (1990), Yang and Zhang (2000), Brandt and Jones (2006), Martens and van Dij (2007), Christenesn and Podolskij (2007).

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forms better than a return-based one. For a comprehensive review on range volatility, see Chou et al. (2015). Other applications of range-based volatility in empirical literature can be found in Miao et al. (2013), Anderson et al. (2015), and Liu et al. (2017).

Given its superiority in estimating volatility, the dynamics of the price range is thus of great interest to both academic researchers and financial practitioners. Chou (2005) proposes a range-based volatility model: the Conditional Autoregressive Range model (henceforth CARR). The CARR model belongs to the family of Multiplicative Error Model in Engle (2002), and provides a nice framework to model the dynamics of range-based volatility. The evolution of the conditional range is specified in a fashion similar to the conditional variance models as in GARCH and is very similar to the ACD model of Engle and Russell (1998). Chou (2005) shows that the CARR model does provide sharper volatility estimates compared with a standard GARCH model in out-of-sample volatility forecasting performed on the S & P500 index.

Efficient estimation requires a proper density specification to the disturbance in the CARR model. A commonly used specification is the Weibull distribution. A CARR model with Weibull distribution is known as WCARR. Despite its success in describing the dynamics of range-based volatility, the WCARR is found to suffer from serious inlier problem: a clear deviation from the Weibull distribution for small range values. This problem has also been mentioned in Chou (2005):

> "*It (the transformed residual) is now much closer to the exponential density function in the sense that the problems of inliers seem to be a lot less serious. There is still however, clear room left for further improvements. This is a potential fruitful topic for future research.*"

In this paper, we conjecture the inlier problem is largely due to a misspecified distribution to the disturbance, and attempt to deal with this problem by specifying a gamma distribution[2]. An interesting property with the gamma distribution is that the distribution of the inlier points in the gamma density can be controlled by adjusting the shape parameter: the larger is the shape the fewer are the inlier points. We expect that this property can be used to reduce the inlier problem. Fig. 1 presents the plots of gamma distributions of different shapes.

To investigate the performance of the conditional autoregressive range model with gamma distribution (henceforth GCARR), we perform empirical studies on a variety of stock indices in different countries over different time periods. We find: (1) the GCARR model shows efficiency gains compared with the WCARR model. The GCARR model reports larger log likelihood function values and smaller AIC and BIC values compared with the WCARR model; (2) The inlier problem is indeed reduced once gamma distribution is used. The empirical results show consistently that the GCARR model suffers less from the inlier problem than the WCARR model; (3) The GCARR model suffers less from the outlier problem than the WCARR model. This is a new finding, which indicates that our gamma distribution also fits the large range values better than the Weibull distribution.

This paper contributes to the existing related studies with at least three significant respects. First, we propose a new CARR model, the GCARR model to describe the dynamics of the range-based volatility. This GCARR model can well reduce the inlier and outlier problems reported by the WCARR. Second, a comprehensive empirical study is performed to demonstrate the superiority of GCARR model over the WCARR model. Last and the most important, the proposed GCARR model can serve as a better benchmark for modelling the range-based volatility.

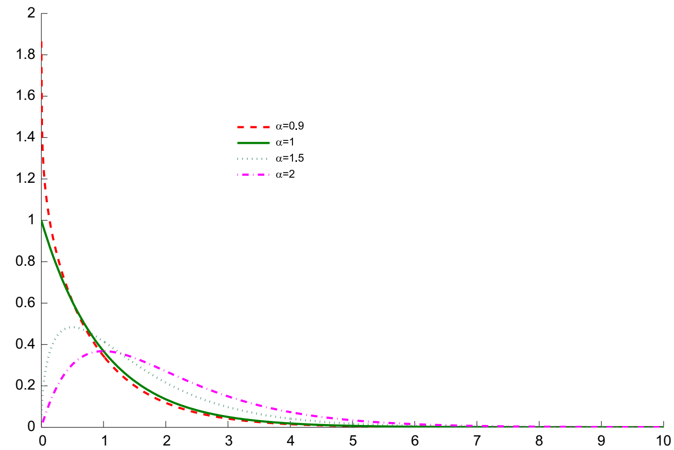

Fig. 1. The density function of gamma distribution of different shapes, $\Gamma(\alpha, 1)$.

The remainder of the paper proceeds as follows. Section 2 presents the GCARR model with some discussions. Section 3 empirically investigates the GCARR model on a variety of stock indices and compares its performance with the WCARR model. The main findings are presented. We conclude in Section 4.

2. Model specification and discussion

In Chou (2005), a CARR model of order (p, q), *CARR(p, q)* is presented as follows:

$$R_t = \lambda_t \varepsilon_t \lambda_t = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{j=1}^{q} \beta_j \lambda_{t-j},\ \varepsilon_t | I_{t-1} \sim f(.\ ),$$

where λ_t is the conditional mean of the range based on all information up to t. The non-negative disturbance term ε_t, is assumed to be distributed with a density function $f(.)$ of a unit mean. The coefficients $(\omega, \alpha_i, \beta_j)$ in the conditional mean equation are all positive to ensure positive range. R_t is the price range defined over time interval [t-1, t]:

$$R_t = max_\tau P_\tau - min_\tau P_\tau,\ \tau \in [t-1, t].$$

where P_τ be the logarithmic price of a financial asset.

2.1. GCARR model

A conditional autoregressive range model of order (p, q) with gamma disturbance is presented as follows:

$$R_t = \lambda_t \varepsilon_t,\ \lambda_t = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{j=1}^{q} \beta_j \lambda_{t-j},\ \varepsilon_t \sim \Gamma(\alpha, 1),\ i.\ i.\ d$$

where $\Gamma(\alpha, 1)$ is the density function of a gamma distribution, and ε_t is assumed to be independent of λ_t through time t. A *CARR(p, q)* model with gamma distribution is called *GCARR(p, q)*.

Similar to the Weibull distribution[3], gamma distribution also takes the exponential density as a special case. Different from the Weibull distribution, the gamma distribution can not be transformed to exponential distribution so long as the shape α does not equal to 1. An interesting property with the gamma distribution is that the distribution of the inlier points in the gamma density can be controlled by adjusting the shape parameter.

[2] The density function of a gamma distribution is

$$g(x; \alpha, \lambda) = \left(\frac{x}{\lambda}\right)^{\alpha-1} \frac{1}{\Gamma(\alpha)\lambda} e^{-\frac{x}{\lambda}},$$

where α and λ are known respectively as the shape and the scale. The gamma distribution takes the exponential density as a special case when α takes value of 1.

[3] The density function of a Weibull distribution is presented as follows

$$f(x; \lambda, \theta) = \frac{\lambda}{\theta}\left(\frac{x}{\lambda}\right)^{\theta-1} e^{-(x/\lambda)^\theta},\ x \geqslant 0$$

. It is clear from the density function that Weibull distribution takes the exponential distribution as a special case when θ takes value of one.