Economic Modelling 60 (2017) 24–28

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/econmod

Time-varying leads and lags across frequencies using a continuous wavelet transform approach

Yoshito Funashima*

Faculty of Economics, Tohoku Gakuin University, 1-3-1 Tsuchitoi, Aoba-ku, Sendai, Miyagi 980-8511, Japan

A R T I C L E I N F O

Article history:
Received 5 April 2016
Received in revised form
25 August 2016
Accepted 25 August 2016
Available online 28 September 2016

JEL classification:
C49

Keywords:
Time-varying leads and lags
Frequencies
Wavelet
Phase difference

A B S T R A C T

A precise understanding of lead–lag structures in economic data is important for many economic agents such as policymakers, traders in financial markets, and producers in goods markets. To identify time-varying lead–lag relationships across various frequencies in economic time series, recent studies have used phase difference on the basis of a continuous wavelet transform. However, the extant literature includes several conflicting interpretations of phase difference. In this study, we extensively discuss wavelet phase difference, determine its most plausible interpretation, and thus attempt to address gaps in the existing literature. Consequently, this study suggests that some lead–lag results of previous works have been driven by incorrect interpretations of wavelet phase difference.

1. Introduction

It is well recognized that lead–lag relationships exist in economic data, hence understanding them is important for policymakers and other economic agents. For practical purposes, in particular, it is essential for many economic agents to identify the leading, coincident, and lagging indicators of business cycles in order to predict their duration (see, e.g., Nefti, 1979).

In fact, a substantial number of papers have hitherto examined lead–lag relationships between key macroeconomic and financial variables. For example, Dekle et al. (2001) examine the relationship between exchange rates and interest rates using high-frequency data from Korea, and Alsakka and ap Gwilym (2010) investigate lead–lag relationships in sovereign ratings.

Recently, many authors have started utilizing wavelet methods to capture the time-varying leads and lags across frequencies (see, e.g., Aguiar-Conraria and Soares, 2014).[1] To be precise, in the growing body of wavelet literature, previous researchers have used phase difference as a tool to obtain information about changing lead–lag dynamics at specific frequencies, such as those in business cycles.

This paper contributes to the literature in the following respects. First, we identify previous works that use wavelet phase difference to analyze lead–lag relationships and demonstrate that wavelet phase difference has been subjected to multiple interpretations. Second, and most important, we investigate the most plausible interpretation and thus attempt to address the gaps in the existing literature. Consequently, this study suggests that some lead–lag results of previous works have arrived at an incorrect conclusion due to the incorrect interpretation of wavelet phase difference.

The remainder of the paper is structured as follows. In Section 2, after a brief explanation of wavelet phase difference, we indicate that different interpretations coexist in the literature. In Section 3, we deliberate on which interpretation should be considered plausible. Section 4 concludes the paper.

2. Different interpretations of wavelet phase difference

To begin with, we summarize the different interpretations of wavelet phase difference in the literature.

* Corresponding author. Tel.: +81 22 721 3355.
E-mail address: funashima@mail.tohoku-gakuin.ac.jp

[1] Recently, an increasing number of studies have used wavelet methods to conduct empirical analysis in the field of economics. See, for example, Aguiar-Conraria and Soares (2011a, 2011b, 2014), Aguiar-Conraria et al. (2012), Rua (2012, 2013), Chen et al. (2013), Trezzi (2013), Fidrmuc et al. (2014), Tiwari et al. (2014), Berdiev and Chang (2015), Cascio (2015), Dima et al. (2015), Jiang et al. (2015), Li et al. (2015), and Aloui et al. (2016).

http://dx.doi.org/10.1016/j.econmod.2016.08.024
0264-9993/

Given a time series $x(t)$, the continuous wavelet transform is given by

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t)\tilde{\psi}^*_{\tau,s}(t)dt, \tag{1}$$

where $\tilde{\psi}$ represents wavelet daughters, s is the scaling factor controlling wavelet length, τ is the translation parameter controlling wavelet location in time, and the asterisk denotes complex conjugation. Note that if the absolute value of s is less (more) than 1, the wavelet is compressed (stretched). Wavelet daughters $\tilde{\psi}$ are obtained by scaling and shifting the mother wavelet ψ:

$$\tilde{\psi}_{\tau,s}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right), \quad s, \tau \in \mathbb{R}, \quad s \neq 0. \tag{2}$$

In line with many other previous studies, we consider the Morlet wavelet, one of the most widely used mother wavelets,

$$\psi_{\omega_0}(t) = \pi^{-1/4}\left(e^{i\omega_0 t} - e^{-\omega_0^2/2}\right)e^{-t^2/2}, \tag{3}$$

where i denotes an imaginary unit (i.e., $i = \sqrt{-1}$) and ω_0 controls the number of oscillations within the Gaussian envelope. Following earlier studies, we assume that $\omega_0 = 6$, because in this case, s is almost equal to the Fourier period.

From the above wavelet transform, one obtains the phase angle

$$\rho_x(\tau, s) = \tan^{-1}\left[\frac{\text{Im}\{W_x(\tau, s)\}}{\text{Re}\{W_x(\tau, s)\}}\right], \tag{4}$$

where $\text{Re}(W_x)$ and $\text{Im}(W_x)$ are the real and imaginary parts of the wavelet transform W_x, respectively. The phase angle indicates the oscillation position of the time series $x(t)$ at a specified time and frequency.

For the bivariate case, we consider two time series of interest, $x(t)$ and $y(t)$. For each wavelet transform, the cross-wavelet transform is given by

$$W_{xy}(\tau, s) = W_x(\tau, s)W_y^*(\tau, s). \tag{5}$$

In order to evaluate the relationship between the two series, we utilize the following phase difference from the phase angle of the cross-wavelet transform:

$$\rho_{xy}(\tau, s) = \rho_x(\tau, s) - \rho_y(\tau, s) = \tan^{-1}\left[\frac{\text{Im}\{W_{xy}(\tau, s)\}}{\text{Re}\{W_{xy}(\tau, s)\}}\right], \tag{6}$$

with $\rho_{xy} \in [-\pi, \pi]$.

As regards the sign of the correlation between $x(t)$ and $y(t)$, to the best of our knowledge, all previous studies without exception interpret the phase difference ρ_{xy} as follows: When $\rho_{xy} \in (-\pi/2, \pi/2)$, $x(t)$ and $y(t)$ move in phase (positive correlation), whereas when $\rho_{xy} \in (\pi/2, \pi) \cup (-\pi, -\pi/2)$, $x(t)$ and $y(t)$ move out of phase (negative correlation). In particular, if $\rho_{xy} = \pi$ or $\rho_{xy} = -\pi$, they move in anti-phase.

However, as for lead–lag relationships, the literature presents completely different interpretations. First, most previous works in the wavelet literature adopt the following interpretation.[2]

Interpretation 1. *If* $\rho_{xy} \in (0, \pi/2) \cup (-\pi, -\pi/2)$, *then* $x(t)$ *leads* $y(t)$. *If* $\rho_{xy} \in (-\pi/2, 0) \cup \rho_{xy} \in (\pi/2, \pi)$, *then* $y(t)$ *leads* $x(t)$.

Second, some studies adopt an interpretation opposite to Interpretation 1.[3]

Interpretation 2. *If* $\rho_{xy} \in (0, \pi/2) \cup (-\pi, -\pi/2)$, *then* $y(t)$ *leads* $x(t)$. *If* $\rho_{xy} \in (-\pi/2, 0) \cup \rho_{xy} \in (\pi/2, \pi)$, *then* $x(t)$ *leads* $y(t)$.

Finally, to the best of our knowledge, two studies, that is, Marczak and Gómez (2015) and Marczak and Beissinger (2016), present the following interpretation.

Interpretation 3. *If* $\rho_{xy} \in (0, \pi)$, *then* $x(t)$ *leads* $y(t)$. *If* $\rho_{xy} \in (-\pi, 0)$, *then* $y(t)$ *leads* $x(t)$.

However, since the above interpretations are provided without any clear explanation, one does not understand why the previous studies present different interpretations.

3. Discussion and illustrations

In this section, we deliberate on which interpretation can be considered plausible and attempt to explain the difference between the three interpretations. The process of our deliberation is as follows. First, as regards the discrepancy in the interpretations when $\rho_{xy} \in (-\pi/2, 0) \cup (0, \pi/2)$, a comparison of Interpretations 1 and 3 with Interpretation 2 shows Interpretation 2 to be inappropriate. Second, as regards the discrepancy when $\rho_{xy} \in (-\pi, -\pi/2) \cup (\pi/2, \pi)$, a comparison of Interpretation 1 with 3 shows that only Interpretation 1 is plausible.

Further, we also discuss the indicators of composite index (CI) in Japan. One reason for choosing Japan is that all the data of the leading, coincident, and lagging indicators are available for roughly the last half century.[4]

3.1. Interpretations 1 and 3 versus Interpretation 2

When $\rho_{xy} \in (-\pi/2, 0) \cup (0, \pi/2)$, we find that Interpretations 1 and 3 differ from Interpretation 2. In view of the difference, one can readily disprove Interpretation 2. Now, consider a simple example of data generated:

$$x_t = \begin{cases} \cos\left(\frac{2\pi}{12}t + \frac{\pi}{3}\right) + \varepsilon_t, & (t \leq 36) \\ \cos\left(\frac{2\pi}{12}t - \frac{\pi}{3}\right) + \varepsilon_t, & (t > 36) \end{cases} \tag{7}$$

and

$$y_t = \cos\frac{2\pi}{12}t + \varepsilon_t, \tag{8}$$

where ε_t is i.i.d. $N(0, 1)$. We give our observations in Panel A of Fig. 1; x leads y by $\pi/3$ for $t \leq 36$ at a 12 cycle, whereas y leads x for $t > 36$. The phase difference ρ_{xy} calculated for 11–13 cycles is displayed in Panel B of Fig. 1.[5] For $t \leq 36$, the phase difference is between 0 and $\pi/2$ (in the vicinity of $\pi/3$). On the other hand, for $t > 36$, the phase difference lies between $-\pi/2$ and 0 (in the vicinity of $-\pi/3$). This simple exercise supports Interpretations 1 and 3.

[2] For Interpretation 1, see Aguiar-Conraria et al. (2012, 2013), Caraiani (2012a), Trezzi (2013), Aguiar-Conraria and Soares (2014), Sousa et al. (2014), Cascio (2015), Funashima (2015, 2016a, 2016b), Ko and Lee (2015), Li et al. (2015), Lin et al. (2016), Dewandaru et al. (2015, 2016), Fousekis and Grigoriadis (2016), and Su et al. (2016). A recent study by Funashima (2016a, Fig. 2) provides a graphic explanation supporting Interpretation 1.

[3] For Interpretation 2, see Aguiar-Conraria et al. (2008), Aguiar-Conraria and Soares (2011b), Caraiani (2012b), Tiwari (2013), Andrieş et al. (2014), Tiwari et al. (2015a, 2015b), and Klarl (2016).

[4] Data are obtained from the website of the Cabinet Office for the Government of Japan. We use the ASToolbox provided by Luis Aguiar-Conraria and Maria Joana Soares to compute the phase difference. The ASToolbox can be downloaded at http://sites.google.com/site/aguiarconraria/joanasoares-wavelets.

[5] Note that the values are median over scales for 11–13 cycles.