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Efficient estimation of macroeconomic equations with unobservable states

Stephen D. Morris

Bowdoin College, Department of Economics, 9700 College Station, Brunswick, ME 04011, United States

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ABSTRACT

Macroeconomic equations, such as the consumption Euler equation, New Keynesian Phillips curve, and Taylor rule, are regularly estimated on an individual basis. However, such relations also jointly determine equilibrium, which may contain unobservable states. This paper shows how to utilize such an equilibrium model to improve the efficiency of individual estimators. In comparison with existing related approaches, this simple framework lends itself naturally to modern medium scale dynamic stochastic general equilibrium models. Not only does the derived estimator exhibit smaller asymptotic variance than equation-by-equation GMM, it also tends to be less prone to small sample distortions from weak identification.

1. Introduction

Anyone who has attempted to bring a macroeconomic equation to the data can attest to the problem of theoretically insensible estimates. This class of equations consists of many specifications of perennial interest to academics and policymakers alike, including the consumption Euler equation, New Keynesian Phillips curve, and Taylor rule. One may find, for example, that the Phillips curve has no slope, therein brining into question the primitive assumption of correct specification. In part, these observations may be due to large standard errors, themselves perhaps an artifact of the relatively short time series commonly utilized. In response to this concern, this paper presents a simple framework for improving the efficiency of estimators for individual macroeconomic equations, and reports some promising simulated and empirical results. For example, using post-War U.S. data and conventional generalized method of moments (GMM), one would conclude that the slope of the Phillips curve is insignificant. But the framework utilized in this paper suggests it is positive at all meaningful significance levels. This has direct and substantive implications for evaluating tradeoffs in the conduct of monetary policymaking.

In order to come to these conclusions, the framework presented in this paper appeals to the fact that individual quantities like the Phillips curve are inherently related to equilibrium, by which we mean some broader model of the interconnected economy. The observation that estimating many equations simultaneously may lead to improved efficiency is no new development. Moreover, the question of improving the efficiency of estimators for macroeconomic equations in particular has been considered in some depth with respect to reduced form auxiliary models, such as vector autoregressions (VARs). But the key trait yet to be tackled is that modern models of aggregate equilibrium – dynamic general equilibrium (DSGE) models, like (Smets and Wouters, 2007) – usually contain many latent, i.e. unobservable states.[1] This usually implies no finite order VAR representation exists (Ravenna, 2007). If DSGE models are deemed accurate depictions of the economy, then not accounting for such latent states is problematic; it not only brings the efficiency of estimators into question, but consistency itself. Yet, accounting for these latent states is no intuitive task, for which we have no straightforward intuition from the literature. The intention of this paper is to bridge that gap.

This paper is structured as follows: Section 2 elaborates further on background and places the paper in the literature. Section 3 considers a motivating example without latent states. Section 4 carries this forth to the general case where latent variables may be present. Section 5 presents the main simulation results, in consideration of the consumption Euler equation, New Keynesian Phillips curve, and Taylor rule, each a single equation from the Smets and Wouters (2007) model. Section 6 provides a robustness check with respect to the full information maximum likelihood estimator. Section 7 considers an empirical application to U.S. historical data. Section 8 concludes.

2. Background

Consider the multiple equation model $y_{im} = \boldsymbol{x}'_{im}\boldsymbol{\beta}_m + \varepsilon_{im}$, equations $m = 1,\ldots,M$ and observations $i = 1,\ldots,I$. The $k \times 1$ predetermined

E-mail address: smorris@bowdoin.edu.

[1] Recent work in DSGE estimation includes Kamber et al. (2016), Pop (2016), and Razafindrabe (2016).

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variables $\boldsymbol{x}_{im}$ are orthogonal to $\boldsymbol{\varepsilon}_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iM})'$, a mean zero error, for each m. Thus kM orthogonality conditions

$$E(\boldsymbol{x}_{im} \otimes \boldsymbol{\varepsilon}_i) = \mathbf{0} \tag{1}$$

for $\otimes$ the Kronecker product. Zellner's (1962) enduring point is that unless all M equations are truly "unrelated," in the sense that the variance-covariance matrix $E(\boldsymbol{\varepsilon}_i \boldsymbol{\varepsilon}'_i)$ is diagonal, the efficient estimator for β_m is not equation-by-equation ordinary least squares (OLS). Rather, it is the seemingly unrelated regression (SUR) estimator, which accounts for such interrelations.

Now consider two sets of orthogonality conditions ubiquitous in macroeconomic literature.

$$\text{Consumption Euler equation:} \quad E(\boldsymbol{Z}_t \varepsilon_t) = \mathbf{0} \tag{2}$$

$$\text{Equilibrium model:} \quad E(\boldsymbol{Z}_t \otimes \boldsymbol{U}_t) = \mathbf{0} \tag{3}$$

$t = 1, \ldots, T$ is a time index. (2) arises from a consumption Euler equation with scalar error ε_t and $\boldsymbol{Z}_t$ a $\zeta \times 1$ vector of valid instruments (Hall, 1978). Hansen and Singleton (1982) utilized these orthogonality conditions in illustration of generalized method of moments (GMM) (Hansen, 1982). The interim has seen many extensions to this basic idea, a research agenda partially summarized by Ludvigson (2013). In contrast, (3) arises from a dynamic stochastic general equilibrium (DSGE) model, with $n \times 1$ mean zero error $\boldsymbol{U}_t$. Ruge-Murcia (2007) provides a meticulous accounting of GMM estimator properties in this alternative setting. In practice, there has been little if any direct overlap between the two research agendas considering (2) versus (3), with the former having salient applications primarily in asset pricing, and the latter in monetary policy analysis. Note, any such estimator based on (2) alone is, in the context of also (3), by definition an equation-by-equation estimator.

And yet, because the Euler equation characterizes household consumption in essentially all DSGE models, ε_t and $\boldsymbol{U}_t$ are not uncorrelated. Indeed, ε_t is one element of a $n \times 1$ vector of structural shocks $\boldsymbol{\varepsilon}_t$, which is related to $\boldsymbol{U}_t$ by the $n \times n$ matrix $\boldsymbol{D}$.

$$\boldsymbol{U}_t = \boldsymbol{D}\boldsymbol{\varepsilon}_t \quad \boldsymbol{\varepsilon}_t = [\ldots \ \varepsilon_t \ \ldots]' \tag{4}$$

With this in mind, simply stack (2) and (3), and compare the result with (1).

$$E(\boldsymbol{Z}_t \otimes [\varepsilon_t \boldsymbol{U}'_t]') = \mathbf{0} \tag{5}$$

Because ε_t and $\boldsymbol{U}_t$ are correlated – and hence not "unrelated" – no GMM estimator based upon (2) alone is efficient. Rather, the GMM estimator which jointly depends upon (3) always has smaller asymptotic variance. In this sense, the consumption Euler equation is only seemingly unrelated from the other equations determining equilibrium.

While the Euler equation provides a convenient pedagogical device for communicating this concept, it is not the only equation of individual interest also related to equilibrium. For instance, the New Keynesian Phillips curve and Taylor rule are also regularly estimated on an individual basis. Of course, the observation that additional moment conditions may serve to improve efficiency of estimators for any single equation is well known, if not at this point self evident. This owes to a voluminous and established literature considering just such augmented GMM estimators (e.g. Newey and McFadden, 1994). But while intuition for adding moment conditions may prove straightforward in other contexts, the same may not be said for macroeconomic quantities such as these, seeing as the additional moments under consideration arise from some DSGE model. Undoubtedly, there are at least four concerns surrounding such an approach.

Prominent concerns regarding DSGE estimation and inference.

1. **Indeterminacy:** (Lubik and Schorfheide, 2004) The DSGE model may have multiple solutions in regions of parameter space, complicating estimation.
2. **Identification:** (Canova and Sala, 2009) DSGE models are subject to numerous identification concerns which are not easily ameliorated.
3. **Misspecification:** (Waggoner and Zha, 2012) There exists no general consensus as to what constitutes correct specification in the DSGE arena.
4. **Latent variables:** (Gallant et al., 2015) GMM is not well understood in the case that there are latent variables, as is almost always the case with respect to DSGE models.

If any of concerns (1.)-(3.) are realized, then any purported improvement in efficiency garnered from a DSGE model towards a single equation is irrelevant; the resulting estimator is no longer even consistent. With respect to point (4.), GMM estimation of models with latent variables has only recently received attention. Thus, the issues at hand are potentially more subtle and insidious than is commonly appreciated. As policymakers now venture to obtain sharper estimates for parameters of paramount importance, such as the slope of the Phillips curve, the otherwise seemingly trivial pursuit of augmenting estimators for single macroeconomic equations with equilibrium conditions plainly demands further attention.

Inasmuch, the purpose of this paper is not merely to reiterate the known fact that adding moment conditions may improve efficiency of estimators. Instead, it is to propose how this may be achieved in a cogent manner for macroeconomic equations specifically, given the four above concerns. Towards this end, the arguments herein deal with the autoregressive parameters from the vector autoregressive moving average (VARMA) solution of DSGE models – an $r \times 1$ vector $\boldsymbol{\phi}$ – rather than the mapping to the $k \times 1$ structural parameters $\boldsymbol{\theta}$. Regarding Concern #1, this reframing placates all concerns related to determinacy of solutions, since the VARMA parameters may be directly estimated without any solution necessary. Regarding Concern #2, one need not be concerned with the identifiability of the entire set of structural parameters, since $\boldsymbol{\phi}$ are trivially identifiable from the Yule-Walker equations. Regarding Concern #3, relaxing these parametric restrictions also addresses misspecification. One need not assume that any DSGE model is the true data-generating process (DGP), but merely that some VARMA model is. Regarding Concern #4, the VARMA model at the same time explicitly accounts for latent states. This assumption is strictly weaker than the commonly held one that a vector autoregression (VAR) is the DGP, which may lead to truncation biases (Ravenna, 2007).

To make these results concrete, this paper considers the well-known (Smets and Wouters, 2007) model, which has VARMA(3,2) representation (Morris, 2016b). The results from post-War U.S. data are as follows: First, with respect to the Euler equation, confidence intervals for the inverse intertemporal elasticity of substitution are multiples smaller utilizing this estimator, compared with classic equation-by-equation GMM. Second, standard errors for the slope of the Phillips curve with respect to marginal costs make otherwise insignificant estimates significantly positive, and consistent with conventional macroeconomic theory. Third, while standard errors of the inflation response of monetary policy in the Taylor rule are made smaller, they still imply the coefficient is statistically insignificant, lending credence to the thesis of changes in regime over this period (Lakdawala, 2016). These conclusions suggest the estimator is empirically useful.

Moreover, the estimator developed in this paper also addresses challenges intrinsic to GMM estimation unrelated from efficiency. Even when instruments are valid, they may prove weak, causing conventional Gaussian asymptotic results to yield poor approximations even in relatively large samples (See Bound et al., 1995). This outcome has been documented in both the cases of equation-by-equation GMM estimation of the Euler equation (Stock and Wright, 2000) and the Phillips curve (Kleibergen and Mavroeidis, 2009). With respect to the Euler equation, the problem's source is best described by the empirical reality that finding instruments which are correlated with the future