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# Unique and multiple equilibria in a macroeconomic model with environmental quality: An analysis of local stability☆

Azam Chaudhry[a,*], Hafsa Tanveer[a], R. Naz[b]

[a] Department of Economics, Lahore School of Economics, Lahore 53200, Pakistan
[b] Centre for Mathematics and Statistical Sciences, Lahore School of Economics, Lahore 53200, Pakistan

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ABSTRACT

This paper develops and analyzes an economic growth model which incorporates environmental quality into the production and utility functions. We solve our model for the balanced growth path and find that a unique low growth equilibrium is attained when environmental quality is given less weight in the utility function. Multiple equilibria exist if environment quality is given greater weight in the utility function. We also perform local stability analysis of our model. We conclude that an economy in which the environmental quality is given relatively less importance by the agents will be caught in low growth, high consumption poverty traps as is the case for many developing countries while other economies can potentially reach a relatively low consumption, high growth steady state if they place greater weight on environmental quality. Finally we look at how the gap between low and high growth equilibria shrinks when individuals place greater weight on environmental quality and how governmental policies can promote growth when societies give less weight to the environment.

## 1. Introduction

Even though economic growth remains the major priority for many economists and policy makers, there has been a growing recognition that environmental degradation is an important developmental issue. The impact of environmental quality on economic growth has been analyzed in both the empirical and theoretical literature. The empirical approach is based on the Environmental Kuznets Curve which theorizes an inverted U-shaped relationship between economic growth and environmental quality. The rationale behind this idea is that a country initially invests in capital and technology that increases growth at the expense of environmental quality; only after a certain level of income is achieved can a country allocate a sufficient amount of capital to abatement activities which leads to an improvement in environmental quality (see Fernández et al., 2012) Because of this, there is general acceptance of the theory that if a country wants to improve environmental quality then at first more attention should be given to growth which will result in an overall increase of income per capita (see Stern et al., 1996).

The theoretical approach used to analyze the relationship between economic growth and environmental quality is based on macroeconomic growth models which incorporate environmental quality and are solved to obtain balanced growth paths. Dinda (2005) combines the stock of capital, the level of pollution, and the stock of environment (natural capital) in an endogenous growth model. Models by Greiner (2005) and Economides and Philippopoulos (2008) incorporate environmental quality into endogenous growth models with public infrastructure expenditure. Gupta and Barman (2010) studied the impact of environmental quality on health and infrastructures facilities and established a direct link between health, capital, infrastructure and environmental quality. Bovenberg and Smulders (1995) and Pérez and Ruiz (2007) create endogenous growth models with pollution augmenting production while Ricci (2007) and Nguyen-Van and Pham (2013) analyze growth models which incorporate environmental quality in their production and utility functions respectively. Itaya (2008), on the other hand, analyzes an endogenous growth model with environmental taxation. Most of the theoretical results in the literature support the idea that the government should divide its budgetary expenditures between infrastructures development, public health and abatement activities because low environmental quality will have a negative impact on economic growth and on public health capital.

Our growth model incorporates environmental quality into the production and utility functions and we use this model to examine the local dynamic properties. The main focus of the paper is to explain local dynamics around each balanced growth path equilibrium when there are multiple long run equilibria and we build on the work of other

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* Corresponding author.
*E-mail address:* Azam@lahoreschool.edu.pk (A. Chaudhry).

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authors like Hosoya (2012) and Gaspar et al. (2014) who have investigated the local dynamics of an endogenous growth model with a public health infrastructure and obtained multiple equilibria. In most of the economic growth models, transitional dynamics have been studied by five different approaches, namely phase diagrams (Romer, 1986), local stability analysis (Benhabib and Perli, 1994), numerical methods (Mulligan and Sala-i-Martin (1993)), explicit dynamic methods Xie (1994) and closed form solutions see (e.g., Boucekkine and Ruiz-Tamarit, 2008; Chilarescu, 2011; Naz et al., 2014, 2016). For this paper we have adopted a local stability analysis approach.

This paper contributes to the literature in multiple ways: First, we develop a macroeconomic model which incorporates environmental quality into the production and utility functions. In our model, a household maximizes its utility function subject to constrained capital stock and exogenously determined environmental quality. Second, we look at a dynamical system which incorporates capital and environmental quality into a neoclassical model and along the balanced growth path we will investigate the possibilities of unique and multiple equilibria. In the case of multiple equilibria, we find that as the weight that individuals put on environmental quality increases, the gap between the low and high growth equilibria shrinks. Third, we perform local stability analysis. Finally, we look at how growth is impacted by increasing the government's environmental quality improving expenditures and how this impact varies depending on the weight that individuals place on environmental quality in their utility functions.

The paper is organized in following manner: In Section 2 we will develop a dynamical system representing a neoclassical growth model with physical capital and environmental quality. In Section 3 we will present the decentralized equilibria and the results of our numerical simulations. In Section 4, we will study the stability of the equilibria and the results of further numerical simulations. In Section 5 we look at the impact of the government's environmental policies on growth and in Section 6 we will present concluding remarks.

## 2. A model with environmental quality

In this section, we develop an economic growth model with physical capital and environmental quality which we will then use to study the equilibria and dynamics of the system.

### 2.1. Production function

The economy is characterized by a standard Cobb-Douglas production function in which we are using three inputs, physical capital, labour and environmental quality, for the production of output:

$$Y(t) = K(t)^{\alpha}[E(t)L(t)]^{1-\alpha},\ 0 < \alpha < 1, \tag{1}$$

where $Y(t)$ is the aggregate output in the economy, $K(t)$ is the amount of physical capital, $L(t)$ is the total labor force and $E(t)$ is a stock variable which measures environmental quality. We also assume that there is no population growth.[1] In our model $E(t)$ is not a choice variable in the optimization problem since individual agents in the economy take the overall stock of the environment as given in their production function (see Dinda (2005)). In the production function, $\alpha$ is the weight assigned to the physical capital (which is also known as the output elasticity of production with respect to physical capital) and $(1-\alpha)$ is the weight assigned to environmental quality.

### 2.2. Evolution of physical capital and environmental quality

Assume that the depreciation rate of physical capital is zero and the government imposes a tax, $\tau$, on output and after tax production (income) which it uses to invest in environmental quality. Then the equation of evolution for the physical capital stock is

$$\dot{K} = (1-\tau)K^{\alpha}E^{1-\alpha} - C,\ \ 0 < \tau < 1,\ \ K(0) = K_0, \tag{2}$$

where $C$ is the consumption. The investment on environment quality is equal to the tax collected by the government and is given by $I_E = \tau Y$.

Environmental quality is one of the key variables in this paper and every agent in the economy takes the environmental quality as given social capital, so the representative household takes $E$ as an exogenous variable. For this reason a joint concavity condition on $C$ and $E$ is not required here. The change in environmental quality over time depends upon the magnitudes of emissions and abatement activity and in this economy the equation of motion for environmental quality is (see Gupta and Barman, 2010)

$$\dot{E} = \tau Y - \theta Y,\ \ 0 < \theta < 1, \tag{3}$$

where $\tau$ is the abatement expenditure rate and is defined as the ratio of abatement expenditure to national income, $\tau Y$ is the abatement expenditure and $\theta$ is the constant emission-output coefficient. It should be noted that a constant emission-output coefficient is inconsistent with the theorized inverse U-shaped environmental Kuznets curve, and the justification for this is that we are focusing on the case of developing countries which lie on the beginning of the curve where there is a fairly constant and positive relationship between emissions and output. The environment constraint shows how the quality of the environment changes over time due to factors such as expenditures to improve environmental quality and environment degrading emissions both of which can vary across countries. These factors can have a significant impact on environmental quality and growth and we later analyze this impact using our model.

## 3. Determination of the equilibria

In this section we set up and solve the dynamic optimization problem for this model. The model and many of the derivations in this section are also present in the work of Hosoya (2012) and Gaspar et al. (2014) who investigated the local dynamics of an endogenous growth model with public health infrastructure and obtained multiple equilibria.

### 3.1. The dynamic system

Let $\rho$ be the constant discount factor, $\gamma$ be the weight of environmental quality in the utility function and $\sigma$ be the inverse of intertemporal elasticity of substitution so that the dynamic optimization problem for the representative agent in this model is[2]

$$Max \quad \int_0^{\infty} \frac{(CE^{\gamma})^{1-\sigma} - 1}{1-\sigma} e^{-\rho t}dt,\ \ \gamma \geq 0,\ \sigma > 0,\ \sigma \neq 1,\ \rho > 0, \tag{4}$$

subject to constraint (2). The details of the dynamic optimization problem are provided in Appendix A. The utility function is concave in consumption $C$. Environmental quality, $E$, is an exogenous variable so we do not require a joint concavity condition of the utility function on $C$ and $E$. Also, the constraint given in (2) is jointly concave in $K$ and $C$. So the conditions of the Mangasarian (1966) sufficiency theorem are satisfied for our model and thus the first order conditions are in fact sufficient.

By introducing two new variables, $X=C/K$ (jump variable) and $Z=K/E$ (state variable), the three dimensional system for $K$, $E$ and $C$, given in Appendix A, reduces to a system of the following two differential equations in terms of $X$ and $Z$:

[1] We normalize the total labor force to unity, $L(t) = 1$, so that all the variables are expressed in per capita amounts.

[2] This type of utility function is similar to the ones used by other authors (e.g., Agénor (2008, 2010), Greiner (2005), Hosoya (2012 and 2014), Gaspar et al. (2014)).