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Robust random effects tests for two-way error component models with panel data

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ABSTRACT

In this paper, two test statistics are constructed respectively for individual and time effects in linear panel data models by comparing estimators of the variance of the idiosyncratic error at different robust levels. The resultant tests are one-sided, and asymptotically normally distributed under the null hypothesis. Power study shows that the tests can detect local alternatives that differ from the null hypothesis at the parametric rate. Due to the first difference and orthogonal transformations used in the construction of variance estimators of the idiosyncratic error, the two proposed tests are robust to the presence of one effect and the possible correlation between the covariates and the error components when the other one is tested. Monte Carlo simulations are carried out to provide evidence on the finite sample properties of the tests.

1. Introduction

In econometric analysis of panel data, the random individual and time effects are usually used to capture the heteroscedasticity of individual and time points. In practice, however, people often don't know whether the random effects exist or not, therefore this can lead to the misspecification of the random effects. As a result, some tests for the existence of the two random effects are essential when we use the panel data models with random effects. Breusch and Pagan (1980) proposed several Lagrange multiplier (LM) tests for the existence of two random effects by testing whether their variances are zero or not, which are widely used in both the theory and the application. Given that variances are nonnegative, one-sided tests for such a problem should be more reasonable and powerful than two-sided ones of Breusch and Pagan (1980). Correspondingly, many one-sided tests and their modified versions have been developed for the existence of random effects, e.g. Honda (1985, 1991), Moulton and Randolph (1989), Baltagi et al. (1992), King and Wu (1997), etc. Moreover, Bera et al. (2001) suggested some simple tests based on the ordinary least square (OLS) residuals for random individual effect in the presence of serial correlation. Most of these tests require the normality assumptions of the random effects and the idiosyncratic error, which cannot be guaranteed in practice. Also note that Bera et al. (2001) did not consider the potential time effect which can distort the size of the tests. Montes-Rojas (2010) extended the method of Bera et al. (2001) to spatial panel models. The reader may refer to an excellent monograph by Baltagi (2008) for a detailed review of the existing random effects tests.

Recently, there has been increasing interest in the tests' robustness to the misspecification of various assumptions, conditions and model settings (Bera et al., 2001). Wu and Zhu (2011) proposed two robust random effects tests for the linear panel data models, which are based on the artificial autoregression modelled by the pairwise differenced residuals over the individual and time indices. The simulation results in this paper show that the tests of Wu and Zhu (2011) obtain the robust properties at the cost of low power. Wu and Li (2014) proposed several moment-based tests for the individual and time effects in panel data models. As Wu and Li (2014) argued, these tests have the desired properties as follows. The tests are very simple and easy to compute; the tests for individual effect are robust to the existence of time effect and the possible correlation between the covariates and the error components; the tests for time effect are also robust to the existence of individual effect and the possible correlation between the covariates and the error components. However, since the centering transformation is used in both the construction of the individual effect tests and the determination of the *p*-values

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of the time effect tests in real applications (see Wu and Li, 2014, Sections 2 and 3), the tests of Wu and Li (2014) cannot be extended to the cases with unbalanced panels. Moreover, since the time effect tests of Wu and Li (2014) are not standard, one needs to center the covariates so that the resultant tests can be performed with p-values of critical values calculated from the standard chi-squared distribution with $T-1$ degrees of freedom. These motivate us to develop some new random effects tests as alternatives for panel data models.

The main contribution of this paper is as follows. To avoid distributional assumptions on the random effects and the idiosyncratic error, we consider a robust method to test for the existence of random effects in linear panel data models. Specifically, we compare two estimators of the variance of the idiosyncratic error at different robust levels, and construct the tests for the existence of random effects in the panel data models with no distributional assumptions. The resultant tests are one-sided, and asymptotically normally distributed under the null hypothesis, which is partly different from those of Wu and Li (2014). Power study shows that the tests can detect local alternatives that differ from the null hypothesis at the parametric rate. Due to the first difference and orthogonal transformations used in the construction of variance estimators of the idiosyncratic error, the two proposed tests are robust to the presence of one effect and the possible correlation between the covariates and the error components when the other one is tested. Moreover, the resultant tests don't need any pretreatment of the data so that the condition $Q'EX_i = 0$ holds (see Wu and Li, 2014, p. 572). In addition, the new tests proposed in this paper can be easily modified to test for the existence of random effects in unbalanced panel data models. The above two properties are main different points between the new tests in this paper and the tests of Wu and Li (2014).

The rest of this paper is organized as follows. In the next section, we introduce some notations and simply describe the involved higher order moment estimation of the idiosyncratic error. In Section 3 , we construct test statistics for the existence of random effects, which are based on the difference of two estimators of the variance of the idiosyncratic error at different robust levels. In this section, the asymptotical behavior of the test statistics is investigated theoretically. In Section 4 , Monte Carlo simulation experiments are carried out for illustration. Some conclusions are given in Section 5 . Proofs of theorems are postponed to the Appendix.

2. Model and notations

Consider the linear panel data model with two-way error components

$$y_{it} = \alpha + X_{it}'\beta + \mu_i + \lambda_t + u_{it}, \quad i = 1,2,\dots,n, t = 1,2,\dots,T, \tag{1}$$

where α is the intercept term, X_{it} is the it-th observation on K covariates, and β is the K-dimensional vector of coefficients of covariates. And, μ_i is the individual effect with zero mean and finite variance (hereafter $\sigma^2_{\mu_i}$) and λ_t is the time effect with zero mean and finite variance (hereafter $\sigma^2_{\lambda_t}$). The variances of random effects μ_i and λ_t are allowed to be heterogeneous so that the model settings are more general. The idiosyncratic error u_{it} varies with individual and time, which is assumed to be independent and identically distributed. The covariates $\{X_{it}, t = 1,2,\dots,T\}$ are independent and identically distributed across individuals, and predetermined, i.e. $E(X_{it}u_{is}) = 0$ for $s \geq t$. We allow non-zero correlation among the random effects μ_i, λ_t and the covariates X_{it} (see, e.g., Hsiao, 2003, Mundlak, 1978), which is often omitted in the existing random effects tests in the literature. Moreover, when focusing on the existence of one effect, we don't need any informations of the other one. In addition, it is worthwhile to point out that the asymptotic results in this paper are based on the setting that the individual number n goes to infinity and time length T is fixed, which is widely used in the literature (Wu and Li, 2014).

As a necessary step, the parameter estimation should be considered firstly. Although there exist more efficient estimators when the effects are not misspecified, we prefer to use the robust within estimator on the misspecification of the random effects,

$$\hat{\beta} = \left[X'\left(I_n - \frac{J_n}{n}\right) \otimes \left(I_T - \frac{J_T}{T}\right) X\right]^{-1} X'\left(I_n - \frac{J_n}{n}\right) \otimes \left(I_T - \frac{J_T}{T}\right) y, \tag{2}$$

where $y = (y_{11}, y_{12}, \dots, y_{1T}, y_{21}, y_{22}, \dots, y_{2T}, \dots, y_{n1}, y_{n2}, \dots, y_{nT})'$, $X = (X_{11}, X_{12}, \dots, X_{1T}, X_{21}, X_{22}, \dots, X_{2T}, \dots, X_{n1}, X_{n2}, \dots, X_{nT})'$, and I_l is an identity matrix of dimension l, J_l denotes a $l \times l$ matrix of ones, and "$\otimes$" denotes the Kronecker product. As Wu and Li (2014) argued, under some regularity conditions, it holds that, regardless of the presence of individual and time effects,

$$\sqrt{n}(\hat{\beta} - \beta) \xrightarrow{D} N(0, \Sigma_1^{-1}\Sigma_2\Sigma_1^{-1})$$

as $n \to \infty$, where $\Sigma_1 = E\left[X_i'\left(I_T - \frac{J_T}{T}\right)X_i\right] - EX_i'\left(I_T - \frac{J_T}{T}\right)EX_i$ and $\Sigma_2 = E\left[(X_i - EX_i)'\left(I_T - \frac{J_T}{T}\right)u_iu_i'\left(I_T - \frac{J_T}{T}\right)(X_i - EX_i)\right]$ with $X_i = (X_{i1}, X_{i2}, \dots, X_{iT})'$ and $u_i = (u_{i1}, u_{i2}, \dots, u_{iT})'$. See Wu and Li (2014) or Baltagi (2008) for more details on the within estimator.

As argued in the Introduction, most of the tests for random effects are Lagrange multiplier tests which need the assumptions of normal distribution of the effects and the idiosyncratic error. Wu and Li (2014) proposed several moment-based tests, which don't need distributional assumptions on the error component disturbances. This paper will propose some new moment-based tests as alternatives. Note that Wu and Su (2010) used the first difference over the individual index and the orthogonal transformation over the time index to wipe out the possible time and individual effects and constructed the second order moment estimator (hereafter $\hat{\gamma}_2^u$) and the fourth order moment estimator (hereafter $\hat{\gamma}_4^u$) of the idiosyncratic error. Clearly, the resultant moment estimators are robust to the misspecification of random effects, and they will play a key role in the construction of test statistics for individual and time effects in this paper. In order to save space, we only give the expressions of the two moment estimators $\hat{\gamma}_2^u$ and $\hat{\gamma}_4^u$ of the idiosyncratic error u_{it} as follows (Wu and Su, 2010, p. 1935),

$$\begin{aligned}\hat{\gamma}_2^u &= \frac{1}{n(T-1)}\sum_{j=1}^{m} M_2'\left(\Delta y_{2j} - \Delta X_{2j}\hat{\beta}\right)^{(2)} \\ &= \frac{1}{n(T-1)}\sum_{j=1}^{m}\left(\Delta y_{2j} - \Delta X_{2j}\hat{\beta}\right)'\left(I_T - \frac{J_T}{T}\right)\left(\Delta y_{2j} - \Delta X_{2j}\hat{\beta}\right),\end{aligned} \tag{3}$$

$$\hat{\gamma}_4^u = \frac{1}{nc_0}\sum_{j=1}^{m} M_4'(\Delta y_{2j} - \Delta X_{2j}\hat{\beta})^{(4)} - 3(\hat{\gamma}_2^u)^2\left[\frac{2(T-1)}{c_0} - 1\right], \tag{4}$$

where $m = [\frac{n}{2}]$ ($[a]$ being the integer part of a, hereafter), $\hat{\beta}$ is the within estimator of Eq. (2), $c_0 = \sum_{l=2}^{T}\frac{l^2-3l+3}{l(l-1)}$, $M_2 = \sum_{j=2}^{T} q_j^{(2)}$, $M_4 = \sum_{j=2}^{T} q_j^{(4)}$, $q_j = \frac{1}{\sqrt{j(j-1)}}[(j-1)e(j) - \sum_{k=1}^{j-1} e(k)]$, $j = 2,3,\dots,T$, with $e(k)$ standing for the k-th column vector of the identity matrix I_T. Moreover, $\Delta y_{2j} = (\Delta y_{2j,1}, \Delta y_{2j,2}, \dots, \Delta y_{2j,T})'$, $\Delta X_{2j} = (\Delta X_{2j,1}, \Delta X_{2j,2}, \dots, \Delta X_{2j,T})'$ and "Δ" stands for the difference operator over the individual index, i.e. $\Delta y_{2j,t} = y_{2j,t} - y_{2j-1,t}$. Hereafter, we denote $\xi^{(k)} = \underbrace{\xi \otimes \cdots \otimes \xi}_{k}$ for any vector or matrix ξ. Under some moment conditions, the above two estimators are respectively consistent and asymptotically normally distributed. The reader can refer to Wu and Su (2010, p. 1935–1936) for more details.