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Optimal reinsurance policies with two reinsurers in continuous time

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ABSTRACT

An optimal reinsurance problem of an insurer is studied in a continuous-time model, where insurance risk is partly transferred to two reinsurers, one adopting the expected-value premium principle and another one using the variance premium principle. The insurer aims to select an optimal reinsurance arrangement to minimize the probability of ruin. To provide an easy-to-implement solution to the problem, (semi)-explicit expressions for the optimal reinsurance strategies as well as the minimal ruin probabilities are derived for several claims distributions. Numerical studies including a real-data example based on the Danish fire insurance losses are provided to illustrate the solution of the problem. Our empirical results based on the Danish data reveal that the heavy-right-tailedness of claims distributions has a significant impact on the optimal reinsurance strategies and has a quite pronounced impact on the residual risk described by the minimal ruin probability.

1. Introduction

Reinsurance is widely used in the insurance industry to transfer and manage risk. It has long been an important topic in both the theory and practice of actuarial science, in particular in non-life insurance mathematics. Straub (1988) provided an excellent exposition on reinsurance (see Chapter 4 therein). There is a large literature on the use of mathematical models to study reinsurance. In particular, an optimal reinsurance problem is widely studied in the literature. Some of the recent works are, for example, Asmussen (2000), Asmussen et al. (2000), Azcue and Muler (2005), Bai et al. (2013), Cai and Wei (2012), Centeno (2005), Choulli et al. (2003), Eisenberg and Schmidli (2009), Grandell (1991), Liang and Guo (2008), Promislow and Young (2005), Meng and Zhang (2010), Meng and Siu (2011a), Meng and Siu (2011b), Meng and Siu (2011c), Meng et al. (2013), Meng et al. (2016a), Meng et al. (2016b), Hipp and Taksar (2010), Schmidli (2001) and Taksar and Markussen (2003), among others. It seems that many of the previous studies may mainly focus on the situation where a reinsurer uses the expected value premium principle. However, it is known that the expected value premium principle cannot take into account the variability of insurance losses. To articulate this shortcoming of the expected value premium principle, some authors, for example Zhou and Yuen (2012) and Chi (2012), considered the variance premium principle to study optimal reinsurance problems in a dynamic and static modeling settings, respectively.

In practice, for some reinsurance arrangements, there may be different lines of business or different reinsurers applying different coverage levels and premium pricing mechanisms. For a discussion on various reinsurance arrangements in practice, one may refer to, for example, Straub (1988). Recently, in a static one-period risk modeling set-up, Chi and Meng (2014) considered the situation where a reinsurance treaty involves two reinsurers with different premium principles. The minimal value at risk (VaR) and conditional value at risk (CVaR) of an insurer's total risk exposure were considered in Chi and Meng (2014). In a continuous-time set-up, Meng (2013) studied an optimal impulse dividend control problem with two reinsurers who calculate premiums by variance premium principles with different parameters. Furthermore, Meng et al. (2016a) extended Meng (2013) to the situation with multiple reinsurers. Inspired by these works, we shall study an optimal risk control problem, where the risk of an insurer is partially transferred to two reinsurers applying different premium calculation premiums, one adopting the expected value premium principle and another one using the variance premium principle. The objective of the insurer is to select an optimal

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reinsurance strategy so as to minimize its probability of ruin. In some parametric cases for the claims distribution, (semi)-explicit solutions to the optimal reinsurance problem are obtained. Numerical examples are provided to illustrate the optimal solutions. A real data example based on Danish fire insurance losses data is presented to illustrate the potential practical application of the model and the impact of heavy-tailedness in the optimal strategies and the minimal ruin probability. The present paper may be related to some recent papers on optimal dividend and reinsurance problems, for example, Meng et al. (2016a) and Meng et al. (2016b). In Meng et al. (2016a), an optimal reinsurance-dividend problem in the presence of multiple reinsurers adopting variance premium principles with different parameters was considered. The objective in Meng et al. (2016a) was to maximize the expected discounted dividends before the time of ruin in the presence of fixed costs and taxes. In Meng et al. (2016b), an optimal reinsurance-dividend problem was also considered in the presence of two types of insurance claims whose premiums were determined by the expected-value premium principle and the variance premium principle. Both the fixed and proportional transaction costs were incorporated in Meng et al. (2016b). The objective in Meng et al. (2016b) was also the maximization of the expected discounted dividends until the time of ruin. Though some mathematical techniques used in the present paper may be related to those in Meng et al. (2016a) and Meng et al. (2016b), there may be some differences between the present paper and the two papers Meng et al. (2016a) and Meng et al. (2016b). For example, the objective function in the present paper is to minimize the probability of ruin of the insurer while, in both Meng et al. (2016a) and Meng et al. (2016b), the objective was to maximize the expected discounted dividends until the time of ruin. The present paper does not consider optimal dividend payments as well as the presence of fixed and proportional transaction costs while the two papers Meng et al. (2016a) and Meng et al. (2016b) did. The present paper considers the situation where one reinsurer applies the expected value premium principle and another reinsurer uses the variance premium principle. Whereas, in Meng et al. (2016a), multiple reinsurers used the variance premiums with different parameters. Though the model considered in the present paper may be considered more simpler than those in Meng et al. (2016a) and Meng et al. (2016b), (semi)-explicit solutions to the optimal reinsurance problem are obtained in different parametric claims distributions which may make the model easy to use in practice. The present paper may also be related to some earlier works on optimal reinsurance-dividend problems in the literature, for example, Meng and Siu (2011a), Meng and Siu (2011c) and Meng (2013). Meng and Siu (2011a) discussed a combined reinsurance-dividend-reinvestment problem, where the objective was to maximize the expected discounted dividends minus the expected discounted reinvestment until the time of ruin. Meng and Siu (2011c) studied an optimal reinsurance-dividend problem in the presence of fixed and proportional transaction costs, where the objective was to maximize the expected discounted dividends after deduction of equity issuance until the time of ruin. Meng (2013) discussed an optimal reinsurance-dividend problem with two reinsurers adopting the variance premium principles with different parameters in the presence of fixed transaction costs and proportional taxes. The objective in Meng (2013) was to maximize the expected discounted dividend payments. Again the objective functions in these works may not be the same as the one used in the present paper. Furthermore, it does not seem that sufficient attention has been given to numerical implementation and real-data studies of optimal strategies in these works. This may be important if one is interested in implementing the optimal strategies in practical situations. Armed with the derived (semi)-explicit forms of the optimal reinsurance strategies in different parametric claims distributions, the present paper provides numerical implementation as well as real data studies based on the Danish fire insurance losses of the optimal reinsurance strategies. Attention has also been given to studying the impact of heavy-right-tailedness of the claims distributions on the optimal reinsurance strategies and the minimal ruin probabilities using real data. It does not seem that these issues may have been well-addressed in the literature.

The rest of this paper is organized as follows. In Section 2, the modeling framework and the optimization problem of minimizing the ruin probability are presented. In Section 3, it is shown that a combination of the proportional and excess of loss reinsurance is optimal among different reinsurance arrangements. (Semi)-explicit expressions for an optimal reinsurance and the value function for the optimization problem are derived in Section 4. Section 5 gives (semi)-analytical solutions to the optimization problem for four parametric claims' distributions, namely an uniform distribution, an exponential distribution, a Gamma distribution and a Pareto distribution. Some numerical examples including the real data example are also provided to illustrate the solutions of the problem for the four parametric cases. Section 6 gives a summary.

2. Model formulation

A complete, filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathbb{P})$ is considered, where the filtration $\{\mathcal{F}_t\}$ satisfies the usual conditions (i.e., the right continuity and $\mathbb{P}$-completeness). As usual, it is assumed that all stochastic processes and random variables are well-defined on this probability space. The insurance risk model presented here is a kind of the classical Cramér-Lundberg model. It is standard and has been widely considered in the actuarial science literature.

Suppose that the surplus process of an insurance company without reinsurance is described by the non-zero drift compound Poisson process $\{P_t, t \geq 0\}$:

$$P_t = x + pt - \sum_{i=1}^{N_t} Z_i,$$

where $x \geq 0$ is the initial surplus or the initial reserve; A constant premium rate p is charged continuously over time from policyholders; $\{N_t, t \geq 0\}$ is a Poisson process with constant intensity parameter $\lambda > 0$; The sizes of the claims $Z_i, i = 1, 2, \cdots$, which are assumed to be independent of $\{N_t, t \geq 0\}$, are independent and identically distributed (*i.i.d.*) random variables with common distribution function F. Assume the distribution function F is continuous and has a finite mean μ and a finite second moment σ^2.

Furthermore, it is supposed that the insurer charges a premium by the expected value principle with a safety loading $\eta > 0$. That is,

$$p = (1 + \eta)\lambda\mu.$$

To manage and control its risk exposures, the insurer can cede part of the loss from each claim to a reinsurance market. A simple situation is considered here, where there are two reinsurers with different risk attitudes participating in a reinsurance treaty. Reinsurer I adopts the expected value principle with a safety loading $\beta > 0$, while Reinsurer II adopts the variance premium principle with a safety loading $\theta > 0$. Mathematically, to underwrite a risk Y, Reinsurer I charges a premium of $(1 + \beta)\mathbb{E}[Y]$ and Reinsurer II charges a premium of $\mathbb{E}[Y] + \theta\mathbb{D}[Y]$, where $\mathbb{D}[Y]$ denotes the variance of Y. Here the insurer and one reinsurer adopt the expected value premium principle while another reinsurer uses the variance premium principle. The insurer uses reinsurance as a vehicle to transfer insurance risk to the two reinsurers. The heterogeneity in the premium rules adopted by the two reinsurers is incorporated in the modeling framework here. The situation where the two reinsurers adopt the variance premium principles with different parameters was