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Modelling the extreme variability of the US Consumer Price Index inflation with a stable non-symmetric distribution

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ABSTRACT

Stable distributions have interesting properties that make them a versatile tool suitable for modelling a wide range of processes from different scientific fields, from meteorology to computer science and from communications to economic theory. Our objective is to use stable laws to get an insight at the distributional characteristics and behavior of the US Consumer Price Index inflation. Such a descriptive model is essentially an easy to use tool that provides us with useful information about the Index, via its ability to generate series with similar characteristics. Besides using an appropriate non-parametric test, an examination via an ensemble of a large number of simulated series is implemented in order to assess the accuracy of the model. Its capabilities and adaptability make it a useful tool for everyone analyzing processes from the field of economics.

1. Introduction

The normality assumption of economic data is widely used and in many cases it is also well founded, but whenever extreme variability is observed, the question of normality deviation is arising. Stable distributions have been known to be implemented in order to explain the stochastic behavior of such processes and our objective is to investigate whether the nature of the CPI inflation can be considered similar to the stochastic nature of the stable laws.

Our work is an addition to the applications of stable distributions in economics and finance that were published by Fama (1965), Mandelbrot (1963), Mittnik et al. (1998), Rachev and Mittnik (2000), Rydberg (2000) and Tsionas (2002). Analysis regarding the behavior, characteristics, structure and dynamics of the CPI inflation, can be found in Apergis (2011), Baillie and Morana (2012), Benkovskis et al. (2012), Funke et al. (2015) and Galí and Gertler (1999), while some of the numerous inflation forecasting papers that were published, include Duarte and Rua (2007), Freeman (1998), Kichian and Rumler (2014), McAdam and McNelis (2005) and Öğünç et al. (2013).

When we fit a stable distribution to a series with high variability, examining the value of the α parameter ($\alpha \in [0, 2]$) is important in order to quantify and evaluate our suspicions of deviation from the gaussian case. Even a value seemingly close to 2 (i.e $\alpha = 1.8$), marks the existence of a noteworthy greater variability than the one present in normal distributions, as it is the case not only with our CPI inflation series, but also with many other processes studied in economics.

The algorithms available play a crucial role in the applicability of the model and therefore its value and usage to the applied researcher, since the inference and simulation methods can determine the speed, accuracy and the overall efficiency of the endeavor. The (Koutrouvelis, 1980, 1981) method for inference stands out from the various alternatives satisfying all of our specifications. Combined with the standard algorithm for generating stable laws by Chambers et al. (1976) and Weron (1996), provide us with everything we need to identify and reproduce the stable sequence we desire.

Besides addressing these issues, we are also trying to get information about the overall adaptability of our model, not just by implementing an appropriate non-parametric test, but also by using an additional, different approach to the problem. We examine the goodness of fit of the stable distribution to the data via an ensemble of 10,000 simulated series from which the mean Cumulative Distribution Function (CDF) is derived and compared with the CDF from the data. The results clearly indicate a very good fit, with the CDF of the CPI inflation being exceptionally close to the average CDF from our simulations, clearly capturing the extreme values of the data that prompted the investigation about the applicability of the stable laws in the first place.

The dataset used to facilitate this undertaking, consists of the monthly US Consumer Price Index (CPIAUCSL) inflation for the past 20 years (US. Bureau of Labor Statistics).

The paper is organized as follows. In the first section we present the tests used and provide useful information about the stable

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distributions by presenting the tools available for the parameter estimation and the available algorithm for simulation. The next section is devoted to the CPI dataset where we estimate the stable parameters, examine the paths we generated and evaluate the applicability of the stable distribution to this type of data. In the final section the conclusions from the model's implementation are reported.

2. Mathematical ingredients

2.1. Kwiatkowski Phillips Schmidt Shin (KPSS) test

The KPSS test (Kwiatkowski et al., 1992) is one of the standard tests used for testing the existence of a level (or trend) stationary series. In the KPSS test for level stationarity, one estimates the following model:

$$\begin{cases} y_t = \mu_t + u_t \quad \text{and} \\ \mu_t = \mu_{t-1} + e_t, \quad \text{with} \quad e_t \sim iid(0, \sigma_e^2) \end{cases} \tag{2.1.1}$$

where u_t is a stationary process and μ_t is a random walk whose initial value μ_0 serves the role of an intercept (level) around which we will test for stationarity. The null hypothesis of level stationarity is specified as $\sigma_e^2 = 0$ which implies that μ_t is a constant, while the alternative of $\sigma_e^2 > 0$ introduces a unit root in the random walk.

When we are testing the null hypothesis of stationarity around mean, versus the presence of a unit root, the test statistics for the KPSS test is:

$$\text{Test Statistic} = \frac{\sum_{n=1}^{T} S_t^2}{s^2 T^2} \tag{2.1.2}$$

where T is the sample size, s^2 is an estimate of the long run variance and $S_t = e_1 + e_2 + \ldots + e_t$.

2.2. Stable distributions

Definition 2.2.1. A random variable X is said to have a stable distribution if for any positive numbers *A* and *B* there is a positive number *C* and a real number *D* such that:

$$AX_1 + BX_2 \overset{d}{=} CX + D \tag{2.2.1}$$

where X_1 and X_2 are independent copies of X and "$\overset{d}{=}$" denotes equality in distribution (Samorodnitsky and Taqqu, 1994).

An equivalent definition to Definition 2.2.1, is the following:

Definition 2.2.2. A random variable X is said to have a stable distribution if there are parameters $0 < \alpha \leq 2, \sigma \geq 0, -1 \leq \beta \leq 1$ and μ real, such that its characteristic function $\phi(\theta)$ has the following form:

$$\phi(\theta) = E \exp i\theta X$$
$$= \begin{cases} \exp\{-\sigma^\alpha |\theta|^\alpha (1 - i\beta(sign(\theta)) \tan \frac{\pi\alpha}{2}) + i\mu\theta\} & \text{if} \quad \alpha \neq 1 \\ \exp\{-\sigma\,|\theta|\,(1 - i\beta\frac{2}{\pi}(sign(\theta)) \log|\theta|) + i\mu\theta & \text{if} \quad \alpha = 1 \end{cases} \tag{2.2.2}$$

where:

$$sign(\theta) = \begin{cases} 1 & \text{if} \quad \theta > 0 \\ 0 & \text{if} \quad \theta = 0 \\ -1 & \text{if} \quad \theta < 0 \end{cases}$$

The parameter α is the index of stability and the parameters σ, β and μ are unique, with β being irrelevant when $\alpha = 2$.

A random variable X is called *strictly stable* if Definition 2.2.1 holds with $D = 0$. A stable random variable X is called *symmetric stable*, if its distribution is symmetric, that is if: X and −X have the same distribution.

Obviously a symmetric stable random variable is strictly stable.

Since Definition 2.2.2 is characterized by four parameters: $\alpha \in (0, 2]$, $\sigma \in \mathbb{R}_{\geq 0}$, $\beta \in [-1, 1]$ and $\mu \in \mathbb{R}$ we will denote stable distributions by $S_\alpha(\sigma, \beta, \mu)$ and we will write:

$$X \backsim S_\alpha(\sigma, \beta, \mu)$$

When X is a symmetric α stable distribution (meaning $\beta = \mu = 0$), we will write:

$$X \backsim S\alpha S$$

The following graphs represent some characteristic stable densities (Figs. 2.2.1 and 2.2.2).

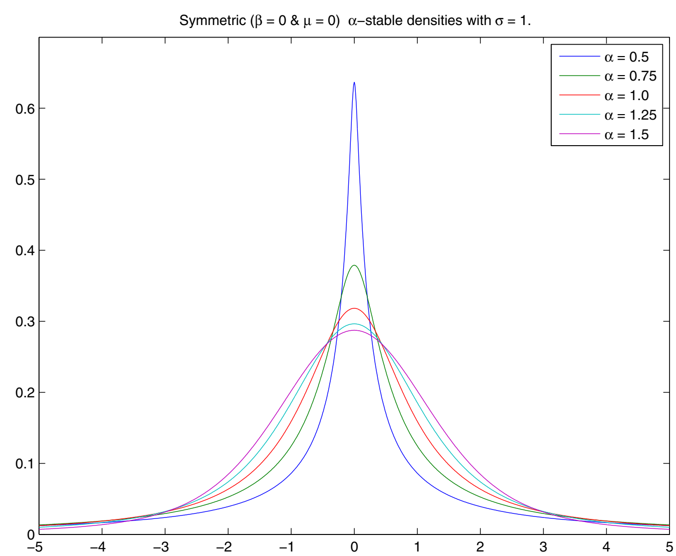

Fig. 2.2.1. Probability density functions for five symmetric ($\beta = 0$ and $\mu = 0$) α-stable random variables. The remaining parameter takes the value: $\sigma = 1$.

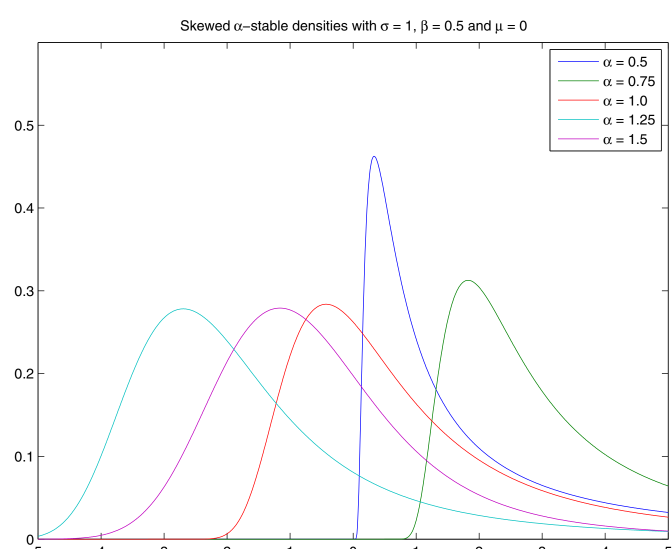

Fig. 2.2.2. Probability density functions for five skewed ($\beta = 0.5 \neq 0$) α-stable random variables. The remaining parameters take the values: $\sigma = 1$ and $\mu = 0$.