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A bivariate Hawkes process for interest rate modeling

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ABSTRACT

This paper proposes a continuous time model for interest rates, based on a bivariate mutually exciting point process. The two components of this process represent the global supply and demand for fixed income instruments. In this framework, closed form expressions are obtained for the first moments of the short term rate and for bonds, under an equivalent affine risk neutral measure. European derivatives are priced under a forward measure and a numerical algorithm is proposed to evaluate caplets and floorlets. The model is fitted to the time series of one year swap rates, from 2004 to 2014. From observation of yield curves over the same period, we filter the evolution of risk premiums of supply and demand processes. Finally, we analyze the sensitivity of implied volatilities of caplets to parameters defining thelevel of mutual-excitation.

1. Introduction

During the recent crisis of European sovereign debts, fixed income markets collapsed and caused liquidity shortfalls in countries of South Europe. The immediacy of information contributed to speed up the tightening of traded volumes of short and long term bonds. And the abrupt decline in demand for debts, due to the anxiety about excessive national debt, even if correlated with a reduction of supply, raised interest rates to historical summits, in Greece (33.7% for the 10 year bond on the 3/2/2012), Italy, Spain and Portugal. On the other side,by the end of 2011, Germany was estimated to have made more than €9 billion out of the crisis as investors flocked to safer but near zero interest rate German federal government bonds. By July 2012 the Netherlands, Austria and Finland also benefited from zero or negative interest rates, as consequence of the high demand for their national debt. This crisis reminds us that interest rates basically depend on the law of supply and demand. There is also compelling evidence that yields of fixed income instruments are affected by liquidity concerns, as shown by Landschoot (2008), Longstaff (2004), Chen et al. (2007), Covitz and Downing (2007) or Acharya and Pedersen (2005). Understanding liquidity effects in bonds markets is then of particular importance for central banks, to define appropriate monetary policy actions.

As liquidity shortages result from a disequilibrium between the global demand and supply for debts, the model developed in this work assumes that the short term rate is ruled by a bivariate Hawkes processes, representing the aggregate bid and ask orders, for fixed income instruments. This approach is fully relevant with the monetary theory as e.g. detailed in the chapter 5 of Mishkin (2007), and presents several interesting features. Firstly, it introduces path dependency and auto-correlation, that are absent from models based on Brownian motions (Brigo and Mercurio, 2007, Cox et al., 1985, Dai and Singleton, 2000, Duffie and Kan, 1996, Hull and White, 1990, Zhang et al., 2015, for a survey), on Lévy processes (Eberlein and Kluge, 2006, Filipović and Tappe, 2008, Hainaut and MacGilchrist, 2010) or on switching processes (Hainaut, 2013; Shen and Siu, 2013). Secondly, it adds mutual excitation and snowball effects, between the supply and demand in interest rate markets. This features is introduced in the dynamics through a bivariate Hawkes process (see Hawkes, 1971a,b; Hawkes and Oakes, 1974). This is a parsimonious self and mutually exciting point process for which the intensity jumps in response and reverts to a target level in the absence of event. As the future of a Hawkes process is influenced by the timing of past events, Errais et al. (2010) use this, combined with a mean reverting drift of the intensity, to generate contagion between defaults in a top down approach to credit risk. Embrechts et al. (2011) apply multivariate Hawkes processes in their analysis of stocks markets. Mutually exciting processes are also used by Aït-Sahalia et al. (2015, 2014), to model two key aspects of asset prices: clustering in time and cross sectional contamination between regions. Bormetti et al. (2015) model systemic price cojumps with a Hawkes factor models. Rambaldi et al. (2015) propose a Hawkes-process approach to explain foreign exchange market activity around macroeconomic news. Zhu (2014) and Hainaut (forthcoming) study affine models with Hawkes jumps. On the other hand, these processes are increas-

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ingly integrated in high frequency finance. Examples include the modeling of the duration between trades (Bauwens and Hautsch, 2009) or the arrival process of buy and sell orders, as in Bacry et al. (2013). Giot (2005), Chavez-Demoulin et al. (2005) or Chavez-Demoulin and McGill (2012) test these processes in a risk management context. Hawkes processes have also been applied to insurance and ruin theory in Dassios and Zhao (2012), Stabile and Torrisi (2010) or Zhu (2013b). A recent survey of the literature on Hawkes processes applied to finance is proposed in Bacry et al. (2015). Hawkes processes also provide an alternative to model based on the theory of extreme values. This approach (see e.g. Embrechts et al., 1997) is by the way applied in a paper of Bali (2007) to explain interest rate volatility.

This research complements the existing literature about interest rate modeling in several directions. It is one of the first to use exclusively a bivariate Hawkes process for the modeling of the term structure of interest rates. Secondly, the model is built on strong economics foundations. Whereas most of existing models, such as developed by Vasicek (1977) and Hull and White (1990) are based on econometrics and are not necessarily reconcilable with the theory of money economics. Another advantage of the proposed bivariate process is that the two factors driving the interest rates can be assimilated to volumes of supply and demand for fixed income assets. As the calibration is easy and robust, these factors can also serve us to monitor the trading activity in interest rate markets, that are mainly "over the counter" and for which we don't have information about exchanged volumes. Thirdly, this work provides all the tools for pricing bonds and for reconciling the dynamics of the short term rate under the real measure, with the term structure of bonds yields, evaluated under the risk neutral measure. We propose a family of changes of measure that preserves the dynamics of the process under real and risk neutral measures. Finally, after an analysis of the dynamics of bond quotes, the moment generating function of bond yields under a forward measure is detailed and a discrete Fourier transform algorithm is proposed to price derivatives.

The paper proceeds as follows. Section 2 introduces the model and derives its main features, like the moments of intensities for the arrival of bid and ask orders. In Section 3, we present equivalent exponential affine measures and study the conditions ensuring that the equivalent measure is risk neutral. After a presentation of the dynamics of the short term rate under this risk neutral measure, a formula for bond pricing is proposed. The Section 4 is about the valuation of derivatives. In Section 5, we fit the model to the time series of one year swap rate. From observation of yield curves over the same period, we filter the evolution of risk premiums of supply and demand processes. Finally, we test the sensitivity of yield curves and smiles of implied volatilities to changes of parameters.

2. Model

The proposed approach to the analysis of interest rate determination looks at supply and demand in the bond market. It finds its foundations in the economics theory as detailed, for example in the chapter 5 of Mishkin (2007) . Each bond price is associated with a particular level of interest rate. Specifically, the negative relationship between bond prices and interest rates means that when a bond price rises its yield falls and vice versa. In economics, the relationship between the quantity supplied and the price is described by the bond demand line. Under the assumption that all other economic variables are held constant, quantities of supplied bonds, noted B_1, increases linearly with bond prices P_1. The supply curve is then described by the following relation:

$$P_1 = L^1 + \theta_1 B_1 \,,$$

where L^1 and θ_1 are respectively the intercept and the elasticity of the supply curve. Under the same assumption, we can derive a demand curve that shows the relationship between the quantity demanded and bond prices. This curve has the usual downward slope found in demand curves, indicating that as the price increases (everything else being equal), the demanded quantity of bonds falls. The equation defining this line is as follows:

$$P_2 = L^2 - \theta_2 B_2 \,,$$

where L^2 and θ_2 are respectively the intercept and the elasticity of the demand curve. The market equilibrium occurs when the amount of demand equals the supply, $B^* = B_d = B_s$, at a price P^* such that,

$$P^* = L^2 - \theta_2 B^* = L^1 + \theta_1 B^* . \tag{1}$$

In economics, a change in market conditions is represented by a parallel shift in the demand or supply curve. Mathematically, this shift corresponds to a modification of the intercept L^2 or L^1. Then, so as to model the dynamics of interest rates, L^1 and L^2 are indexed by the time, t. According to relation (1), the volume of exchanged bonds at any given time is hence equal to

$$B_t^* = \frac{L_t^2 - L_t^1}{\theta_1 + \theta_2}, \tag{2}$$

and the equilibrium bond price at time t, that is inversely proportional to the equilibrium yield is given by

$$P_t^* = \frac{\theta_1}{\theta_1 + \theta_2} L_t^2 - \frac{\theta_2}{\theta_1 + \theta_2} L_t^1 \propto \frac{1}{r_t}. \tag{3}$$

The formation of this equilibrium and the impact of a marginal shock on the demand curve is illustrated in Fig. 1. As the interest rate is inversely proportional to P^*, if the function $\frac{1}{r}$ is approached by a Taylor development of first order, we infer that the equilibrium interest rate, that is denoted by r_t, is proportional to the difference between the intercepts of supply and demand curves, scaled by some positive constants:

$$r_t \propto \frac{\theta_2}{\theta_1 + \theta_2} L_t^1 - \frac{\theta_1}{\theta_1 + \theta_2} L_t^2 . \tag{4}$$

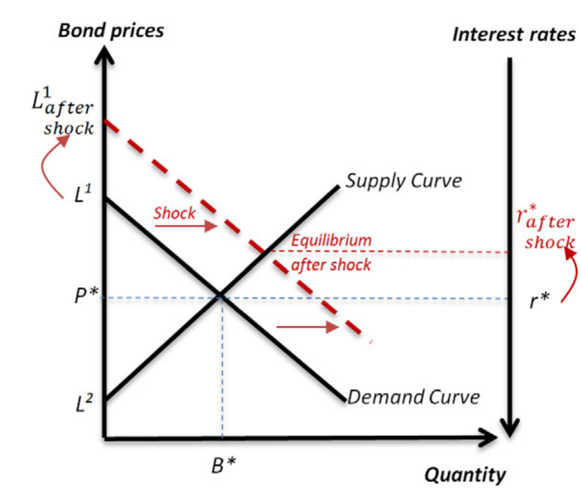

Fig. 1. This graph illustrates the relation between bond prices, interest rates and supply–demand curves. It also shows that a marginal positive shock on the demand corresponds to an increase of the intercept in the demand equation. This positive shock on demand raises bond prices and decreases the equilibrium yield.