Economic Modelling 54 (2016) 260–275

Contents lists available at ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Empirical analysis of stock indices under a regime-switching model with dependent jump size risks

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A R T I C L E I N F O

Article history:
Accepted 23 November 2015
Available online 29 January 2016

Keywords:
Markov regime-switching model
Volatility clustering
Jump risks
Stock index

A B S T R A C T

In this study, we propose a regime-switching model with dependent jump size risks to capture important characteristics of cyclical movements and abnormal shock events. We further demonstrate that the two-state model provides asymmetric and leptokurtic return features, and volatility clustering is observed empirically using 12 years of daily data for the S&P 500, Dow Jones Industrial Average (DJIA), and Nikkei 225 indices. In addition, our results indicate that the regime-switching model with dependent jump size risks is superior to the competing models.

1. Introduction

There has been an accelerating trend in recent decades to create passively managed mutual funds that are based on market indices, such as index funds or exchanged traded funds (ETFs). The stock market index provides reliable market information as well as a better understanding of market forces. It also creates a benchmark against which investors and money managers can measure. The stock market index is a useful tool used by investors and financial managers to describe the market and compare the returns on specific investments. According to the theory and numerous empirical evidence of the Efficient Market Hypothesis (EMH), it is impossible to consistently outperform the market without increasing the risk level. Additionally, a majority of mutual funds fails to outperform the market. Therefore, we can buy into the market through index-related funds with very low management fees. The so-called "index investing" is growing and prevailing not only because it aims to match market performance but also because it incurs very few expenses. There are many developed derivatives of stock indices such as stock index futures and stock index options. The derivatives of stock indices have become important tools with which to hedge risks. Therefore, it is vital to capture the dynamics of the stock market indices. Developing appropriate models to describe their dynamics and trends has drawn increasing attention from individual investors, fund managers, financial companies, researchers and the government.

In the early literatures, stock returns are assumed to follow a traditional geometric Brownian motion, including the Black–Scholes model (BSM), and this assumption is reasonable under relatively stable market conditions. However, the existence of cyclical price movements generates a series of regime-switching models on asset pricing. Hamilton (1989) first proposes the regime-switching model to capture the expansion–recession cycles for the growth rate of Gross National Product. The literature has shown that this model and its variants have been widely applied to analyze economic and financial time series (Bollen et al., 2000; Chang and Feigenbaum, 2008; Chun et al., 2014; Engel, 1994; Engel and Hamilton, 1990; Garcia and Perron, 1996; Goodwin, 1993; Hardy, 2001; Kim and Yoo, 1995; Schaller and van Norden, 1997; Schwert, 1989; Sola and Driffill, 2002).

In the past several decades, significant events including the dot-com bubble in 2000, the September 11 attacks in 2001, the end of the Iraq war in 2003, and the global financial crisis in 2008 occurred, leading to abnormal jumps in stock prices and returns (Lin et al., 2014; Su and Hung, 2011). Unfortunately, the regime-switching model cannot comprehensively describe dramatic changes in such a scenario, and in this paper we propose a regime-switching model with jump size risks to address the jump phenomenon in financial markets. Our model is not the first regime-switching model with jump risks. Elliott et al. (2007) proposed a Markov-modulated jump diffusion model to evaluate the European options. In the model, the market interest rate, jump frequency, mean, and volatility of the underlying asset price change over time according to the state of the economy, which is governed by a continuous Markov chain. In addition, Bo et al. (2010) investigated the same Markov model where the focus was on currency options. In a more recent paper, Chang et al. (2013) provided a closed-form solution for

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http://dx.doi.org/10.1016/j.econmod.2015.11.016
0264-9993/

Table 1
Summary statistics of S&P 500 index return.

	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	Total
Panel A: trading days													
Number of days	251	252	248	252	252	252	252	251	251	253	252	252	3018
Max	0.0347	0.0465	0.0489	0.0557	0.0348	0.0162	0.0195	0.0213	0.0288	0.1096	0.0684	0.0430	0.1096
Min	−0.0285	−0.0600	−0.0505	−0.0424	−0.0359	−0.0165	−0.0169	−0.0185	−0.0353	−0.0947	−0.0543	−0.0398	−0.0947
Mean	0.0007	−0.0004	−0.0006	−0.0011	0.0009	0.0003	0.0001	0.0005	0.0001	−0.0019	0.0008	0.0005	7.88E-06
Std	0.0114	0.0140	0.0136	0.0164	0.0107	0.0070	0.0065	0.0063	0.0101	0.0258	0.0172	0.0114	0.0136
Skewness	0.0598	0.0007	0.0205	0.4251	0.0532	−0.1102	−0.0155	0.1028	−0.4941	−0.0337	−0.0605	−0.2110	−0.1088
Kurtosis	2.8535	4.3882	4.4478	3.6610	3.7589	2.8623	2.8493	4.1553	4.4481	6.6754	4.8510	4.9599	10.2871
Panel B: jump days													
In excess of 2%	14	18	12	23	10	0	0	2	6	31	27	12	155
Mean	0.0241	0.0287	0.0297	0.0314	0.0285	0.0000	0.0000	0.0279	0.0280	0.0409	0.0315	0.0276	0.0310
Std	0.0037	0.0067	0.0084	0.0103	0.0075	0.0000	0.0000	0.0076	0.0070	0.0215	0.0116	0.0072	0.0133
In excess of −2%	9	19	13	29	5	0	0	0	11	41	28	10	165
Mean	−0.0234	−0.0268	−0.0282	−0.0270	−0.0275	0.0000	0.0000	0.0000	−0.0265	−0.0411	−0.0353	−0.0340	−0.0313
Std	0.0026	0.0094	0.0077	0.0056	0.0058	0.0000	0.0000	0.0000	0.0051	0.0209	0.0160	0.0154	0.0135
In excess of ±2%	23	37	25	52	15	0	0	2	17	72	55	22	320
Mean	0.0055	0.0002	−1.88E-05	−0.0012	0.0024	0.0000	0.0000	0.0021	−0.0012	−0.0058	−0.0003	0.0019	−0.0011
Std	0.0239	0.0293	0.0304	0.0303	0.0302	0.0000	0.0000	0.0300	0.0298	0.0459	0.0369	0.0342	0.0340

their Markov-modulated jump diffusion model and empirically confirmed the existence of jump switching and clustering.

Table 1 reports summary statistics of the S&P 500 index returns from 1999 to 2010. In Panel A, the summary statistics are based on daily returns on trading days, and in Panel B, we show samples of large returns (jumps). From Panel A, we can see that the mean return is negative from 2000 to 2002 and in 2008. As is observed, the return volatility is larger in the same years than in other years. This may due to the Internet bubble in 2000 and the financial crisis in 2008. Generally, the dynamics of price and return of the S&P 500 can be classified into two states, expansion and recession. A state of recession is a period of low returns and high volatility, and a state of expansion is a period of high returns and low volatility.

Panel B in Table 1 presents the summary statistics of stock index returns on large return (jump) days where the return is in excess of ±2%.[1] Specifically, Panel B shows the number of jump days, the means, and the standard deviations of jump day returns. Except in 2004 and 2005, jumps appear every year while the jump frequency and the mean and standard deviation of jump day returns are state dependent. The mean frequency of the jumps in the entire period is 26.67, whereas the mean of jump frequencies in the recession state and expansion state is 46.5 and 16.75, respectively. Additionally, the means and standard deviations of jump day returns are higher in the recession state than those in the expansion state. When the information arrives, asset returns not only generate an abnormal jump but the mean and volatility of this jump size also vary under different states. Therefore, the mean and volatility of jump returns are dependent on different states of the economy.

In this paper, we propose a regime-switching model with dependent jump size risks, in which the jump size of the underlying asset changes over time according to the state of the economy for two main reasons. First, the past literature has documented strong empirical evidence of regime-switching behavior of stock market prices (Alizadeh and Nomikos, 2004; Hardy, 2001; Pan and Li, 2013; Rey et al., 2014; Schaller and van Norden, 1997; Schwert, 1989; Timmermann, 2000). Second, empirical observations also show that jump sizes in equity markets are not independent but seem to come together for a certain period. According to the previously observed features on large return days, we empirically find jump clustering, which means that jumps are more frequent in some periods than others, and different jump sizes under different states are also observed. Therefore, we incorporate both jump intensity and state-dependent jump sizes into the regime-switching model.

The regime-switching model with dependent jump size risks has the ability to capture cyclical movements as well as abnormal jump attributes of the underlying asset price. This paper extends the Markov-modulated diffusion model with independent jump risks (Chang et al., 2013; Lin et al., 2014) and empirically examines three stock indices, the S&P 500, DJIA and Nikkei 225 indices. The expectation maximization (EM) algorithm is applied to estimate the parameters of the model while also applying the Supplemented Expectation Maximization (SEM) algorithm to estimate the standard deviation of these parameters. From the empirically estimated parameters in the dynamic model and the derived stock prices, we show that the model is superior to the competing models in stock indices. The estimation results also indicate that our model may capture some critical empirically observed features of asset returns, including asymmetry, leptokurtosis, and volatility clustering. Moreover, the results suggest that jump frequencies and jump sizes are not independent, because high jump size risks are generally followed by continued high jump size risks for the period of the high arrival rate, and low jump size risks are generally followed by continued low jump size risks for the period of the low arrival rate. Therefore, the behaviors of jumps can address jump clustering or volatility clustering driven by jump frequencies and jump sizes.

In this paper, we propose a regime-switching model with dependent jump size risks, in which the jump size of the underlying asset changes over time according to the state of the economy. This paper contributes to the literature on asset pricing and risk management (Chang et al., 2013; Elliott et al., 2007; Elliott et al., 2010; Li et al., 2016; Lin et al., 2014, 2015; Merton, 1976; Su and Hung, 2011). First, we propose a more general jump size risk model, which advances the jump diffusion model to a regime-switching model with dependent jump size risks (RSMDJ) based on a reduced form of the regime-switching model. Second, we develop EM and SEM algorithms to estimate the parameters of the RSMDJ in the past estimation literature of the EM and SEM algorithm (Lange, 1995; Li et al., 2016; Lin et al, 2014; Lin et al., 2015; Mandelbrot and Benoit, 1963; Meng and Rubin, 1991). Finally, actual market data are used to examine the empirical fit performance. Past studies have provided strong empirical evidence of regime-switching behavior in the price in financial markets (Bollen et al., 2000; Chang et al., 2013; Chun et al., 2014; Elliott et al., 2007; Elliott et al., 2010; Garcia and Perron, 1996; Li et al., 2016; Lin et al., 2014; Lin et al., 2015; Rey et al., 2014). Compared to the competing models, our regime-switching model with dependent jump size risks can better explain the dynamics of the S&P 500, DJIA and Nikkei 225 indices. The empirical results are significant in capturing the asymmetry, leptokurtosis, and volatility clustering of stock returns.

The paper is organized as follows. Section 2 outlines the economic framework of the regime-switching model as well as the regime-

[1] We assume that price changes of less than 2% are noise.