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Utility indifference valuation of corporate bond with credit rating migration by structure approach☆

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ABSTRACT

An indifference pricing model for corporate bond with rating migration risk is established in this article. Under the structural framework, the credit rating migration is modeled by the first attempt in an incomplete market, so far as we know. The model results in a HJB system. With the help of the dynamic programming theory, a closed form solution is derived by imposing a condition on the credit rating migration boundary. With the explicit migration boundary and closed form solution, the model is easy to be applied in practice. Based on the pricing formula, the impacts of the parameters on the indifference price are analyzed and relative financial explanations are discussed.

1. Introduction

Corporate bonds are very popular in financial markets. After the Financial Crisis of 2008 and European Debt Crisis of 2010, credit information in corporate bonds was made more interesting to the investors, where the credit rating risks become more and more important. Therefore, it is significant to research into the valuation of the corporate bonds with the probabilities of credit rating migration or insolvency. For insolvency, there are already many research works. In this paper, for an incomplete market, we focus on the indifference valuation of corporate zero-coupon bonds with credit rating migration. In our model, the structural framework is considered to study indifference price with credit rating migration risk. In our knowledge, this is the first attempt to deal with credit rating migration in this way in an incomplete market.

In a complete financial market, claim pricing usually has two equivalent approaches: replication and hedge. A unique and fair price consistent with no-arbitrage can be determined by a perfect replication or hedge. However, in reality, most situations are incomplete. Market frictions such as transactions costs, non-traded assets, and portfolio constraints make perfect replication impossible. In such situations, there is no longer a unique price theoretically. Thus, utility indifference valuation methodology, initiated by Hodges and Neuberger (1989), with the advantage including economic justification and incorporation of risk aversion, is a useful tool to ensure the claim price uniquely in the incomplete market. The idea of utility indifference valuation is to find a price at which the buyer (or writer) of the derivative is indifferent with or without the derivative in terms of maximum utility. Therefore, the approach will lead us to solving portfolio optimization problems in the incomplete market, and we shall use the dynamic programming approach. Liang and Jiang (2012) applied the utility indifference valuation method to corporate bonds pricing with the default probability. The indifference price and hedging strategy are obtained. More papers on indifference valuation can be seen, e.x. in Henderson and Hobson (2004) and Sircar and Zariphopoulou (2007).

Traditional models for defaults are divided into two broad categories: structural and reduced-form (or intensity-based), respectively. In a structural model, the evolution of the firm's value is considered, with the assumption that default occurs when the firm's value falls below some (random or non-random) insolvency threshold. A reduced-form approach concerns exogenous reason rather than the firm's value itself, where the default time is modeled by introducing a hazard rate (default intensity) (see Jarrow and Turnbull, 1995; Lando, 1998; Duffe and Singleton, 1999 and so forth). As the originator of structural models, Merton (1974) presumed that the firm's value process follows a geometric Brownian motion, with regarding the corporate bonds as contingent claims. The default event may only occur at the maturity of the claim. Black and Cox (1976) extended Merton's model by introducing safety covenants that give bondholders the right to recognize a firm if its asset falls below some given threshold, i.e. the first-passage-time model. The key difference is that the Black and Cox model interpreted the observed feature in which the default may occur not only at the debt's maturity, but also prior to this date. A lot of research focuses on the extension of first-passage-time models (see Leland, 1994; Longstaff and Schwartz, 1995; Leland and Toft, 1996; Briys and de Varenne, 1997 and so forth).

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Both structural and reduced form models begin to demonstrate only the credit event of default. For credit rating migrations, most researches mainly adopted transfer intensity matrix. By this way, the reduced form framework is naturally developed for the dynamic process of credit rating migrations (see Jarrow et al., 1997; Das and Tufano, 1996; Lando, 1998, 2000a, 2000b; Thomas et al., 2002 and so forth). Narayan et al., 2004 consider investment grade stocks. By using a panel data model of price discovery, they find that the importance of the CDS market in price discovery improves but the stock market still dominates the price discovery process.

The firm's values play an important role in the credit migrations, which is not mentioned in most application models. Especially it is a weak point of the reduced form model which is assumed that the credit event is independent of the behaviors of the firm itself, while a structural model is better on it. Also by the structural model, the migration boundary is also indicated, by which investor could get more credit information more conveniently. Liang and Zeng (2015) and Hu et al. (2015) started to use the structural model to analyze credit rating migration risk in terms of the firm's value in a complete market. Here, we go further to consider the structure model to price a corporate bond with credit rating migration risk in an incomplete market.

The aims of this article are 1. to understand credit migration in a different way, which has a threshold relative to the firm's value; 2. to use the structural model, which has advantage on hedge and on migration boundary, to price a corporate bond with credit rating migration risk; 3. to study pricing in an incomplete market which has more practice significant; and 4. to find a closed form solution of the bond price. Using this model, it is possible to predict a credit ratio migration by checking the threshold. We define two credit rating grades: high rating grade and a low one by a given credit rating migration threshold. In practice, this boundary can be calibrated from real data by using the maximum likelihood method. When the firm's value is higher than the threshold, we call the firm in the high rating grade, otherwise, it is in the low one. Once the value of the firm crosses the barrier, its credit rating grade changes. Our model's advantage is that it takes the correlation of the firm's value and market stock price into consideration. According to the literatures of the researches on hedge ratios on corporate bonds by structural models, e.x. Schaefer and Strebulaev (2008) and Barsotti and Viva (2015), our model is helpful for hedging purposes.

The rest of the article is constructed as follows. In Section 2, we present the structural model under the utility indifference valuation framework in high grade and low one respectively. Then a corresponding HJB equation system is set by the principle of dynamic programming. The two HJB equations for the high and low grades are connected by a credit rating migration boundary. In Section 3, by imposing an additional credit rating boundary condition, we derive a closed formula under CARA utility functions. Numerical results are presented in Section 4.

2. Modeling

In this section, the structural model is developed for embedding the utility indifference valuation framework. The corresponding HJB equations are set by the principle of dynamic programming in both credit rating grades.

2.1. Assumptions

Assumption 2.1. (the market). Let (Ω, F, P) be a complete probability space. The market is built with three kinds of assets: a risk-free asset (the bank account), corporate stocks and corporate bonds. Let S_t be the value of the stocks at time t which satisfies:

$$\begin{cases} dS_t = \mu_S S_t dt + \sigma_S S_t dW_t^S, \quad t \geq 0, \\ S_0 = s, \end{cases} \tag{1}$$

where μ_S and σ_S represent the yield and the volatility rates of the stocks respectively, $\{W_t^S\}_{t\geq 0}$ is a Brownian motion with its natural filtration $\{F_t^S\}_{t\geq 0}$.

Assumption 2.2. (process of the firm asset with credit rating migration). Let $\tilde{V}_t$ denote the firm's value. It satisfies

$$\begin{cases} d\tilde{V}_t = \left(\mu_{V_1} 1_{\{\tilde{V}_t > \tilde{k}\}} + \mu_{V_2} 1_{\{\tilde{V}_t \leq \tilde{k}\}}\right)\tilde{V}_t dt + \sigma_V \tilde{V}_t dW_t^V, \quad t \geq 0, \\ \tilde{V}_0 = v, \end{cases}$$

where $\mu_{V_i}(i=1,2, \mu_{V_1} > \mu_{V_2})$ represent the surplus of the expected returns of the firm under the high and low credit grades respectively, and σ_V is the volatility. $\{W_t^V\}$ is the Brownian motion which generates the filtration $\{F_t^V\}_{t\geq 0}$. The existence of the solution of above SDE which has a discontinuous drift coefficient is proved by Halidias and Kloeden (2006). The firm's value and the stocks are correlated as

$$\mathrm{Cov}\left(dW_t^S, dW_t^V\right) = \rho dt, \quad 0 \leq \rho < 1.$$

Let $F_t = F_t^V \vee F_t^S$, and the corresponding filtered probability space be $(\Omega, F, \{F_t\}_{t\geq 0}, P)$. Define $\tilde{k} = ke^{rt}$, k is assumed to be a constant, which is an exogenously given value to divide the firm's discount value into two regions to correspond the two grades respectively, i.e. when the firm's discount value excess is k, the firm is in the high rating grade, otherwise it is in the low grade.

In the following, for simplicity, discounted (to time zero) wealth process is considered. Therefore, we denote the discounted variable $V_t = e^{-rt}\tilde{V}_t$. Then, the discounted process is

$$\begin{cases} dV_t = \left\{\mu_{V_1} 1_{\{V_t > k\}} + \mu_{V_2} 1_{\{V_t \leq k\}} - r\right\} V_t dt + \sigma_V V_t dW_t^V, \quad t \geq 0, \\ V_0 = v. \end{cases}$$

Assumption 2.3. (the credit rating migration time). The credit rating migration time is the first moment when the firm's grade changes as follows:

$$\tau_1 = \inf\{t > 0 | V_0 > k, V_t \leq k\}, \quad \tau_2 = \inf\{t > 0 | V_0 < k, V_t \geq k\},$$

where τ_1 and τ_2 denote the first moment of credit downgrade and upgrade respectively.

Remark 2.1. We can choose start time $t=0$ in any life time of the bond, so that, the credit rating migration time can be multiple. In another word, in our model, the upgrades and/or downgrades are allowed to happenmultiple times in the life time of the bond.

Assumption 2.4. (the contract of corporate bonds). A corporate zero-coupon bond is considered. $\tilde{F}$ is its face value and its discounted (to time zero) face value is $\mathrm{F} = e^{-\mathrm{rt}}\tilde{F}$. P_t denotes the discount value of the bond at time i. T is the maturity time. Therefore, an investor can get $P_T = \min\{V_T, \mathrm{F}\}$ at T. This bond has credit rating migration possibility.

Assumption 2.5. (the investors). The investor is assumed to have an CARA utility function: $U(x) = e^{-\gamma x}$. $\gamma > 0$ denotes the risk aversion parameter.

Remark 2.2. We choose this utility function because it is a common one and we obtain a closed form solution with it. For more general and practice utility functions, such as mean–variance one, e.x. used in Westerlund and Narayan (2012) and Phan et al. (2015) can be approximated by this one (see Aivazian et al., 1983).