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Revisiting the long memory dynamics of the implied–realized volatility relationship: New evidence from the wavelet regression☆

Jozef Baruník[a,b,*], Michaela Hlínková[a]

[a] Institute of Economic Studies, Charles University, Opletalova 26, 110 00 Prague, Czech Republic
[b] Institute of Information Theory and Automation, Czech Academy of Sciences, Pod Vodarenskou Vezi 4, 182 00 Prague, Czech Republic

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ABSTRACT

The literature studying stock index options confirms severe biases and inefficiencies in using implied volatility as a forecast of future volatility. In this paper, we revisit the implied–realized volatility relationship with wavelet band least squares (WBLS) exploring the long memory of volatility, a possible cause of the bias. Using the S&P 500 and DAX monthly and bi-weekly option prices covering the recent financial crisis, we conclude that the implied–realized volatility relation is driven solely by the lower frequencies of the spectra representing long investment horizons. The findings enable improvement of future volatility forecasts as they support unbiasedness of implied volatility as a good proxy for future volatility in the long run.

1. Introduction

Option prices are widely believed to carry information relating to expectations of market participants about the future movement of the underlying asset prices in financial markets, mostly its volatility. The volatility implied by an option's price is the forecast of the future return volatility over the remaining life of the relevant option if the option markets are efficient. Early papers studying the phenomenon of implied–realized volatility relation use volatility implied by option pricing models – most commonly Black and Scholes (1973) or Hull and White (1987) – and come to the conclusion that volatility inferred from option markets is a biased predictor of stock return volatility (Day and Lewis, 1992; Lamoureux and Lastrapes, 1993; Canina and Figlewski, 1993; Jorion, 1995).

In contrast, Christensen and Prabhala (1998) and Christensen and Hansen (2002) use a wide variety of methods to show that information content of option implied volatility is superior to that of the past volatility, and it is a less biased (although still biased) predictor of future realized volatility than what has been previously shown. The authors shed new light on the dubiety about the informational content of option implied volatility by specifying the sources of error in previous research. For example, the choice of particular option contracts for extracting volatility and lower liquidity of the option market than in the underlying asset market. Moreover, Christensen and Prabhala (1998) and Christensen and Hansen (2002) find that overlapping data errors can cause cross-correlation in the volatility series, which stems from the overlapping period between the current implied volatility and future implied volatility. In light of these methodological issues, Christensen and Hansen (2002) conclude that option implied volatility is a more efficient forecast for future realized volatility than historical volatility, but it does not subsume all information contained in historical volatility, and it results in upward biased forecasts.

Unlike the traditional concepts using the work of Black and Scholes (1973) or Hull and White (1987) to extract volatilities from options, model-free implied volatility (MFIV) introduced by Britten-Jones and Neuberger (2000) is not based on any specific option pricing model, and it is derived from no-arbitrage conditions. Jiang and Tian (2005) extended the simple measure of implied volatility to all martingale asset price processes and express the formula in forward rather than spot prices. Most notably, Jiang and Tian (2005) first find that the MFIV subsumes all information contained by historical and Black and Scholes (1973) implied volatility and is a more efficient forecast of future realized volatility. Hence, informational content of option implied volatility in the subsequent research is analyzed using the model-free

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* Corresponding author at: Institute of Economic Studies, Charles University in Prague, Opletalova 26, 110 00, Prague, Czech Republic. Tel.: +420 776 259273.
E-mail address: barunik@fsv.cuni.cz (J. Baruník).

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measure.[1] For example, Seo and Kim (2015) find implied volatility to have varying forecasting ability depending on the level of investor sentiment. Although weaker than for stock markets, Chatrath et al. (2015) and Padungsaksawasdi and Daigler (2014) confirm the relationship in commodity markets.

In subsequent research, Andersen and Bondarenko (2007) and Andersen et al. (2015) argue that MFIV computation brings serious practical limitations, yielding inaccurate results. The main problem is the lack of liquid options with strike prices covering the entire return distribution, including its tails. The authors advocate using limited strike ranges at a given point in time instead. The concept is called model-free corridor implied volatility (CIV), previously introduced by Carr and Madan (1998a). While different measures can be obtained, depending on the width and positioning of the strike ranges, Andersen et al. (2015) advocate fixing the range of strikes at a level that provides broad coverage but avoids excessive extrapolation of noisy or non-existing quotes for the out-of-the-money options. Recently, Muzzioli (2013) shed more light on the information content of different parts of the risk neutral distribution of the stock price by considering different corridors in CIV.

When assessing the efficiency of implied volatility forecasts, one needs to have return volatility at hand. However, actual volatility has not been a directly observable variable for a long time. In recent years, as a consequence of the increased availability of high-frequency data, another subject has brought new insight into the implied–realized volatility relationship; the concept of realized volatility. Andersen et al. (2003) and Barndorff-Nielsen and Shephard (2004a) have shown that realized variance provides a consistent nonparametric measure of price variability over a given time interval. An immense literature studying the realized volatility emerged in the past decade discussing the impact of noise as well as jumps in the volatility measurement, concluding that realized volatility is unbiased and a consistent measure of quadratic variation only if we assume no market microstructure noise in the process. The literature also argues that it is important to separate the jump process and use the estimator robust to noise to recover a true underlying volatility. For the measurement of realized volatility, we use one of the most recent jump wavelet two-scale realized volatility estimators (JWTSRV) by Barunik and Vacha (2015), which compares to other estimators used in the literature very well. JWTSRV is able to estimate volatility under the jumps and microstructure noise consistently. The forecasting power of the estimator is studied using a Realized GARCH framework in Barunik et al. (2016).

Using increasingly precise measures, the recent literature suggests that the predictive regression between implied volatility and realized volatility is a cointegrating relationship, and OLS estimation should be avoided as it will result in biased estimates (Bandi and Perron, 2006; Nielsen and Frederiksen, 2011). The relationship is driven primarily by the long memory of both implied and realized volatilities, a key stylized fact commonly found in empirical research across a wide variety of asset classes (Baillie, 1996b; Cont, 2001). Using a spectral method, Bandi and Perron (2006) and Nielsen and Frederiksen (2011) confirm that in a long run, implied volatility is an unbiased predictor of future realized volatility. Still, the results do not say anything about short-term unbiasedness, and they rely on Black–Scholes implied volatility only. Generally, band spectrum regression may be a useful tool in the situation where we believe the relationship between variables is dependent on frequency. The concept was introduced to econometrics by Engle (1974) and further shown to be useful for estimation of cointegrating regressions (Phillips, 1991; Marinucci and Robinson, 2001).

While Bandi and Perron (2006); Nielsen and Frederiksen (2011); Kellard et al. (2010) use a Fourier transform to estimate the relationship in the frequency domain, we contribute to the literature by proposing the band regression on the spectrum estimated by wavelet coefficients. The wavelet transform offers localized frequency decomposition, providing information about frequency components. As a result, wavelets have significant advantages over basic Fourier analysis when the object under study is locally stationary and inhomogeneous — see Gençay et al. (2002); Percival and Walden (2000); Ramsay (2002). This can be a crucial property, as the implied–realized volatility cointegrating relationship may potentially lie in a non-stationary region (Kellard et al., 2010). Wavelets also allow us to study the relationship in the time–frequency domain. We motivate this dynamic by estimating the wavelet coherence measure to study the implied–realized relationship. While wavelet coherence may be used as the “lens” into the relationship that shows the dynamics through time, as well as frequencies at once, a newly proposed wavelet band spectral regression allows us to estimate the relationship.[2]

The contribution of this paper is twofold. First, we emphasize the importance of the implied volatility measure in studying the implied–realized volatility relationship. We compare MFIV and the recently proposed CIV as measures of option implied volatilities with realized volatility and recently proposed jump-wavelet realized volatility (JWTSRV) capable of separating the continuous part of the volatility from jumps as well as noise. We argue that it is crucial to use proper measures for finding the answer to the question of whether the option implied volatility is an efficient forecast of the future realized volatility. Second, we bring new evidence on the unbiasedness of ex-ante implied volatility as a predictor of ex-post realized volatility by allowing long memory dynamics in the time series. We find that the dependence comes solely from the longer time horizons, and when estimated using wavelet band least squares, the implied volatility forecasts are unbiased forecasts of future volatility. These findings greatly improve the understanding of volatility dynamics and add to previous findings of Li (2002), who stress the importance of long memory in studying the implied–realized volatility relationship, or Kinateder and Wagner (2014) who argue that long memory volatility prediction is influenced by the variance term structure.

The methods are applied to the German DAX and U.S. S&P 500 stock market indices covering the 2008 financial crisis, with abrupt changes in prices. Unlike the previous studies, we use both call and put options, and we use options with monthly as well as bi-weekly maturities.

2. Volatility measurement

Consider a univariate risky logarithmic asset price process p_t defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$. The price process evolves in continuous time t over the interval $[0, T]$, where T is a finite positive integer according to a jump diffusion process:

$$dp_t = \mu_t dt + \sigma_t dW_t + \xi_t dq_t, \tag{1}$$

where μ_t is a predictable mean, σ_t a strictly positive volatility process, W_t is standard Brownian motion, $\xi_t dq_t$ is a random jump

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[1] Many studies use the Chicago Board Option Exchange (CBOE) Volatility Index (VIX) as a proxy for model-free implied volatility of S&P 500. Introduced by the CBOE in 1993, its methodology was revised in 2003 using a new model-free measure of expected volatility and thus can be used conveniently.

[2] Note that long memory properties in the emerging markets have been largely studied in the recent literature (Yalama and Celik,2013; Degiannakis and Livada, 2013; Hull and McGroarty, 2014; Charfeddine and Ajmi, 2013)