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# Optimal risk and dividend strategies with transaction costs and terminal value

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ABSTRACT

This paper assumes that an insurance company can control the surplus by paying dividends, raising money and buying proportional reinsurance dynamically. The reinsurance premium is assumed to be calculated via the variance premium principle. Under the objective of maximizing the insurance company's value, we identify the optimal joint strategies and consider the effects of transaction costs and arbitrary terminal value at bankruptcy. From the results, we see that refinancing should be considered if and only if the terminal value and the transaction costs are not too high and the company is on the brink of bankruptcy, and the amount of each capital injection remains constant; the optimal ceded proportion of risk decreases with the current surplus and remains constant when the surplus exceeds some constant level; the optimal dividend distribution policy is of barrier type when the dividend rate is unrestricted or is of threshold type when the dividend rate is bounded, respectively. In particular, the insurance company should declare bankruptcy as soon as possible if the terminal value is high enough.



## 1. Introduction

From the viewpoint of corporate finance, a company's value can be measured by the expected discounted sum of dividend payments until the time of bankruptcy. Determining the optimal dividend strategy for maximizing the company's value is a long standing problem in mathematical insurance. Its origin can be traced as early as the work of De Finetti (1957). Since then much research on this topic has been carried out in varieties of risk models. The question of when to declare dividends and how much of them should be distributed is tricky. Research has shown that, when the dividend rate is unrestricted, the barrier strategy is often optimal. That is, no dividend is paid while the surplus is below a barrier $b \geq 0$ and the overflow with respect to the barrier $b$ is paid out as dividends immediately. Barrier strategy is practical and thus has been widely studied by Asmussen and Taksar (1997), Hϕgaard and Taksar (2004), Lϕkka and Zervos (2008), Kulenko and Schmidli (2008), Belhaj (2010), Yao et al. (2011), Hunting and Paulsen (2013), among others. However, when the considered dividend strategies are restricted to those of bounded dividend rates, the threshold strategy is often optimal. That is, dividends are paid at the maximum admissible rate as soon as the surplus exceeds a certain threshold $u \geq 0$. Some literature on the threshold dividend strategy includes Asmussen and Taksar (1997), Gerber and Shiu (2006), Yao et al. (2014), Zhu (2015) and so on.

It is well known that reinsurance plays an important role in both the theory and practice of insurance risk modeling, by which an insurer can transfer the risks to a second insurance carrier, namely, a reinsurer. A reinsurance contract is said to be “cheap” if the cedent pays the same fraction of the premium as the reinsured. While it is said to be “expensive” if the cedent pays a larger fraction of the premium than the fraction to be reinsured. The excess can be viewed as the transaction cost for a reinsurance contract. One of the typical reinsurance contracts is proportional reinsurance, under which the reinsurer takes a stated percentage share of each policy that an insurer issues. That is to say, the reinsurer will receive that stated percentage of the premiums and accordingly pay the stated percentage of claims. In addition, when the proportional reinsurance is taken as a risk control, the expectation premium principle is commonly used as the reinsurance premium principle due to its simplicity and popularity in practice. Although the variance principle is another important premium principle, few papers consider using it for risk control in a dynamic setting. Generally speaking, the expectation premium principle is commonly used in life insurance which has the stable and smooth claim frequency and claim sizes, while the variance premium principle is extensively used in property insurance. Both dividend and reinsurance are important issues in modeling insurance risk. To maximize the company's value, the insurer needs to balance risk control and dividend payout in terms of reinsurance and dividend distribution policies. Recently, some attention has been paid to the combined optimal dividend and reinsurance problem. As an extension of the classical dividend problem, it assumes that an insurer can control the dividend stream and risk exposure in terms of reinsurance. In the recent past, there have been many articles

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published on this problem under the expectation premium principle, for example, Taksar and Zhou (1998), Hϕgaard and Taksar (1999), Choulli et al. (2003), Cadenillas et al. (2006), Meng and Siu (2011) and so on. Only a few papers investigated this problem under the variance premium principle, see Zhou and Yuen (2012), Yao et al. (2014). Notably, although there are fruitful research results on optimal dividend and reinsurance strategies, very little work considers the problem under a terminal value at bankruptcy, say $P$. The terminal value can be viewed as the salvage value for $P \geq 0$ and the penalty amount for $P < 0$. This problem was firstly brought out in Taksar (2000), in which the company's value was defined as the expected discounted total dividends until the time of bankruptcy and the expected discounted terminal value at bankruptcy. Under the objective of maximizing the company's value, they obtained optimal dividend and reinsurance strategies by using some techniques in stochastic control theory. But the "expensive" reinsurance and the negative terminal value were not taken into account there. Liang and Young (2012) extended this problem by assuming the reinsurance was "expensive" and allowing for an arbitrary terminal value. They obtained the explicit solutions for the optimal dividend and reinsurance strategies by employing the Legendre transform. Other literature on this issue includes Taksar and Hunderup (2007), Xu and Zhou (2012) and Yao et al. (2014).

In addition, the literature mentioned above did not consider the possibility of refinancing. As we know, when an insurance company encounters financial difficulty, it can continue the business by injecting capital. Of course, it requires financing costs, such as the proportional and fixed transaction costs generated by the advisory, consulting and issuance of securities, etc. Up to now, the combined dividend, refinancing and reinsurance problem with transaction costs has been studied extensively. The company's value was usually measured by the expected discounted total dividends minus the expected discounted costs of refinancing until the time of bankruptcy. To maximize the company's value, the insurance company must seek optimal dividend, refinancing and reinsurance strategies. For example, He and Liang (2009) and Barth and Moreno-Bromberg (2014) solved the optimal problem under the expectation premium principle. They also considered the effects of the fixed and proportional transaction costs in refinancing process. Peng et al. (2012) and Guan and Liang (2014) continued to investigate this problem under the assumption of "expensive" reinsurance. However, Zhou and Yuen (2012) solved the problem under the assumption of variance premium principle. The proportional cost and the "cheap" reinsurance were considered in the risk model. Yao et al. (2014) further extended the problem by allowing for the non-negative terminal value and the fixed transaction cost. They first focused on the combined optimization problem of dividend, refinancing and reinsurance with non-negative terminal value. They redefined the company's value as the expected sum of the discounted terminal value and the discounted dividends less the expected discounted costs of refinancing until the time of bankruptcy. Under the assumption of "cheap" proportional reinsurance, they obtained the explicit solutions of optimal strategies in both cases with unrestricted and restricted dividend rates and thus analyzed the effects of proportional and fixed transaction costs. As far as we know, with the exception of Yao et al. (2014), very little work has considered the combined optimal dividend, reinsurance and refinancing strategies with non-zero liquidation value. From the discussed literature, we can see that either the barrier dividend strategy or the threshold dividend strategy is often optimal, depending on whether there exist restrictions on dividend rates; the insurer would buy less reinsurance when the surplus increases; he may refinance when and only when the company is on the brink of bankruptcy and the size of each capital injection keeps constant. The decision to refinance or not depends on the relationships among the model's parameters.

It is worthwhile to note that sometimes the transaction cost for reinsurance contract and negative terminal value at bankruptcy are unavoidable. So we need further research on the optimization problem in the case of "expensive" reinsurance and arbitrary terminal value. Inspired by the above references, we extend the risk model in Yao et al. (2014) by including "expensive" reinsurance and an arbitrary terminal value in this paper. To maximize the insurance company's value, we seek the optimal dividend, refinancing and proportional reinsurance strategies. We solve the problem by using some techniques beyond the Legendre transform in Liang and Young (2012). The explicit solutions are given in both cases with unrestricted and restricted dividend rates, and the effects of transaction costs and terminal value $P \in \mathbb{R}$ are analyzed. The rest of this paper is organized as follows. In Section 2, we use a diffusion approximation of the Cramér–Lundberg model with reinsurance to formulate the optimization problem for a controlled diffusion model with dividend, refinancing and "expensive" proportional reinsurance policies. In Section 3, we first consider two suboptimal problems when the dividend rate is unrestricted. Then we identify the value function and the optimal strategy with the corresponding solution in either category of suboptimal problems, depending on the relationships among the coefficients. In Section 4, we solve the problem when the dividend rate is bounded in a similar way. Finally, we conclude the study in Section 5.

## 2. Model formulation and the optimal control problem

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathrm{P})$ be a probability space, on which all stochastic quantities in this paper are well defined. Here $\{\mathcal{F}_t\}_{t\geq 0}$ is a filtration, which satisfies the usual conditions. In mathematical insurance the surplus of an insurance portfolio is usually described in terms of the Cramér–Lunberg model process $\{Z_t\}_{t\geq 0}$ satisfying

$$Z_t = x + ct - \sum_{n=1}^{N_t} Y_n,$$

where $Z_0 = x$ is the initial surplus, $c > 0$ is the premium rate, $\{N_t\}_{t\geq 0}$ is a Poisson process with constant intensity $\lambda$, random variables $Y_n$'s are positive *i.i.d.* claims with common finite mean $\mu_1 > 0$ and finite second moment $\mu_2^2 > 0$. Under the variance premium principle, it has

$$c = \mathrm{E}\left(\sum_{n=1}^{N_1} Y_n\right) + \theta_1 \mathrm{D}\left(\sum_{n=1}^{N_1} Y_n\right) = \lambda(\mu_1 + \theta_1 \mu_2^2), \tag{2.1}$$

where $\theta_1 > 0$ is a loading associated with the variance. E and D stand for expectation and variance, respectively. Suppose the insurer purchases a proportional reinsurance contract with ceded proportion $a \in [0, 1]$. That is to say, for each claim of size $Y_i$, the insurer covers $(1-a)Y_i$ and the reinsurer covers the rest $aY_i$. Then the total ceded risks up to time $t$ are given by $\sum_{n=1}^{N_t} aY_n$ and the aggregate reinsurance premium under the variance principle is

$$c^a t := \mathrm{E}\left(\sum_{n=1}^{N_t} aY_n\right) + \theta_2 \mathrm{D}\left(\sum_{n=1}^{N_t} aY_n\right) = \lambda\left(a\mu_1 + \theta_2 a^2 \mu_2^2\right)t, \tag{2.2}$$

where $c^a$ is the rate of premiums and $\theta_2 \in (\theta_1, \infty)$ is a loading associated with the variance of ceded risks. Here, the reinsurance is "expensive" due to the condition $\theta_2 > \theta_1$. Then the surplus process in the presence of "expensive" proportional reinsurance can be written as

$$Z_t^a = x + (c - c^a)t - \sum_{n=1}^{N_t} (1-a)Y_n, \tag{2.3}$$

with $Z_0^a = x$. We approximate Eq. (2.3) by a pure diffusion model $\{X_t^a, t \geq 0\}$ with the same drift and volatility. Specifically, $X_t^a$ satisfies the following stochastic integral equation

$$X_t^a = x + \int_0^t \lambda\mu_2^2(\theta_1 - \theta_2 a^2)\mathrm{d}s + \int_0^t \sqrt{\lambda}\mu_2(1-a)\mathrm{d}B_s, \tag{2.4}$$

with $X_0^a = x$.