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Research article

Structured products under generalized kappa ratio

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ABSTRACT

We examine the maximization problem of performance measure of financial structured products. For this purpose, we introduce the kappa ratios, based on downside risk measures which take account of the asymmetry of the return probability distribution. First, we deal with the optimization of some standard structured portfolios. We examine in particular the optimal combination of risk free, stock and call/put instruments with respect to kappa performance measures and in particular to the Sharpe–Omega ratio. Then, we provide the general solution of the optimal positioning problem with respect to kappa ratios. We analyze its properties and compare it to the portfolio profile that is optimal with respect to the standard expected utility criterion.

1. Introduction

Since the 2008 debacle, structured finance has seen many developments which can be split into three main areas: regulatory, investor transparency and finally financial modeling. In this paper, we focus on the latest issue. In fact, the increasing complexity of the structured product market, and the ever growing range of products being made available to investors, invariably creates challenges in terms of efficient assembly, management and dissemination of information.

Investors in financial structured products are generally responsive to the idea to participate to stocks potential upside returns with anticipated acceptable losses on their initial capital. By combining two or more securities, a structured product permits for a variety of risk-return combinations that traditional instruments such as stocks and bonds do not allow for. The product payoff structure is tailored to fit investor's aspirations and needs.

Financial structured product could be defined as "security or other instrument, the return on which is based on the performance of one or more reference assets, which may include stocks, indices, funds, . . . " (source: Structured Product Association – USA).

Financial regulators have increased barrier to entry for money manager hoping to deal with complex product . For example in France for example, the AMF considers that the trading of structured products with complex embedded options are subject to a pre-approval (checks on competency, market access and technology) depending on the manager experience to deal with exotic options.

Historically, there were two main categories of financial structured products: protected equity note and market-linked certificate of deposit (these products were predominately constructed on a call option).

The main objectives of this paper are the search of the optimal payoff and the determination of the risk/reward trade-off according to a generalized performance ratio called kappa measure and to provide the risk profile of the optimal product. In this paper, we focus on plain vanilla portfolio type composed of free risk asset, risky asset and/or options written on it. This structure allows replicating the payoff of standard structured products (by opposition to path-dependent products).

Moreover, we re-examine the optimal positioning feature. Leland and Rubinstein (1976) have introduced the option based portfolio insurance (OBPI). It consists of a portfolio invested in a risky underlying asset S covered by a listed put written on it. Their optimalprofiles

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can be determined from the optimal positioning problem which has been addressed in the partial equilibrium framework by Brennan and Solanki (1981) and by Leland (1980). The portfolio value is a function of the benchmark in a one period set up. Then, the optimal payoff which maximizes the expected utility is determined. It depends crucially on the risk aversion of the investor. Following this approach, Carr and Madan (2001) introduce markets in which exist out-of-the-money European puts and calls of all strikes. This assumption allows examining the optimal positioning in a complete market and is the counterpart of the assumption of continuous trading. More specific insurance constraints can be considered and utility maximization can be solved (see e.g. Bertrand et al. (2001), El Karoui et al. (2005) and Prigent (2006) for quite general insurance constraints). The optimal positioning can be also examined within rank dependent expected utilities (RDEU) as in Jin and Zhou (2008) for the dynamic case and Prigent (2008) for the static case, and also when the investor has a utility with ambiguity aversion as in Ben Ameur and Prigent (2013). Recently Hentati-Kaffel and Prigent (2016) extend the optimal portfolio positioning problem by introducing mixtures of probability distributions to model the log returns of financial assets and provide the general solution for log return with mixture distributions and in a the case of equity portfolio insurance, Bahaji (2014) suggested solution of optimality within a two-equity asset framework in the sense of consumption–investment decision making.

One of the main criticisms of structured products has been the underestimation of assessing risk because of their illiquidity and complexities of performance and risk evaluation. Indeed, payoffs of structured products are non-linear with respect to the underlying asset. This leads to a non-normal returns distribution and for risk profiles similar to those of options. To overcome the non-adequacy of traditional risk and performance measures (such as Sharpe ratio or Jensen alpha) and therefore to provide more adequate optimization solutions, we address our analysis by using alternatives measures. Among these alternative measures, the Omega function has been recently proposed by Keating and Shadwick (2002). It takes account of investor loss aversion, which is in line with results of Tversky and Kahneman (1992) and of Hwang and Satchell (2010) who illustrate the important role of loss aversion in particular for asset allocation problems. Bertrand and Prigent (2011) use the Omega performance measure to compare standard portfolio insurance strategies. They show that the CPPI method provides better results than the OBPI one for "rational" thresholds. Empirical results show that this measure is more stable than other risk measures such as RoCVaR, RoVaR and Sharpe (Hentati et al., 2010, see) but it has many local solutions because of the non-convexity of Omega function. Hentati and Prigent (2011) introduce Gaussian mixtures to model empirical distributions of financial assets and solve the portfolio optimization problem in a static way, taking account of discrete time portfolio rebalancing. More generally, the kappa (n) measures, a generalized downside risk-adjusted performance measure has been provided. This latter is based on n-order lower partial moments as risk measures. In this case, the Sortino and omega ratios are considered as single cases of kappa. The first one is obtained when n=2 and the second for n=1. Farinelli and Tibiletti (2008) develop an integrated decision aid system for asset allocation based on a several performance ratios, in particular the Omega and Sortino one. Zakamouline and Koekebakker (2009) provide an analysis of portfolio performance evaluation with generalized Sharpe ratios. As mentioned by and Hwang and Satchell (1999), the Sortino ratio is linked to an utility function involving lower risk aversion. More generally, Zakamouline (2010) argue that kappa measures is based on piecewise linear plus power utility functions. Darsinos and Satchell (2004) show that n-order Stochastic Dominance implies the kappa $(n-1)$ dominance.

The major contribution of this paper is to derive general conditions to achieve the maximum by using the kappa ratios for different types of structured product. According to Henriksson and Merton, (1981), Dybvig and Ingersoll (1982) results, it is well known that the Sharpe ratio can be manipulated by option-like strategies. In this context, Goetzman et al. (2002) determine portfolio strategies which maximize the Sharpe ratio and they prove that appropriate combinations of puts and calls lead to significantly higher Sharpe ratios than "linear" portfolios. The approach adopted in this paper is quite similar, except that we use the kappa ratios instead of the Sharpe ratio itself. For this purpose, in a first step, we consider a portfolio manager who invests in three basic assets: a risk-free market account, denoted by B, a risky asset (equity), denoted by S and Call/Put written on this equity. In a second step, our aim is to maximize and analyze the optimal portfolio positioning with respect to kappa ratios under given constraints. We begin by determining the necessary conditions to determine precisely the downside risk component. Subsequently, we study the minimization problem of the put component under the constraint of fixed expectation.

The paper is organized as follows: Section 2 is a reminder of the definitions and the main properties of the kappa measures. Section 3 deals with the kappa ratio maximization for linear combinations of the three basic assets. Section 4 provides the general solution of the optimal positioning in financial derivatives with respect to kappa ratios. We prove that, unlike the result of Goetzman et al. (2002) related to the Sharpe ratio maximization, the payoff of the optimal structured portfolio is not always increasing and concave. It can correspond for instance to a straddle. This result is in line with previous results about portfolio optimization within rank dependent utility, as in Prigent (2008).

2. Kappa performance measures

The Omega measure is based on the portfolio return values below and above a given threshold. It is defined as the probability weighted ratio of gains to losses relative to a return threshold. The Omega measure is compatible with the second order stochastic dominance. This measure can potentially take account of the whole probability distribution of the returns. It requires no parametric assumption on the distribution and is equal to

$$\Omega_L(X) = \frac{\int_L^b (1 - F(x))\, dx}{\int_a^L F(x)\, dx}, \tag{1}$$

where $F(.)$ is the cdf of the random variable X (for example equal to the portfolio return) defined on the interval $[a, b]$. The level L is the threshold chosen by the investor: returns smaller than L are viewed as losses and those higher than L are gains. Thus, for a given threshold L, the investor would prefer the portfolio with the highest Omega measure.

As shown by Kazemi et al. (2004), the Omega function is equal to

$$\Omega_L(X) = \frac{\mathbb{E}_{\mathbb{P}}\left[(X-L)^+\right]}{\mathbb{E}_{\mathbb{P}}\left[(L-X)^+\right]}. \tag{2}$$

This is the ratio of the expectations of gains above the given level L upon the expectation of losses below . Therefore, $\Omega_F(L)$ can be interpreted as a ratio call/put defined on the same underlying asset X, with strike L and computed with respect to the historical probability $\mathbb{P}$. The put correspond to the risk measure component. It allows the control of the losses below the threshold L.

Kazemi et al. (2004) define the Sharpe–Omega by

$$\text{Sharpe–Omega} = \Omega_L(X) - 1 = \frac{\mathbb{E}_{\mathbb{P}}[X] - L}{\mathbb{E}_{\mathbb{P}}\left[(L-X)^+\right]}. \tag{3}$$

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