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An efficient estimate and forecast of the implied volatility surface: A nonlinear Kalman filter approach☆

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ABSTRACT

As suggested by numerous studies, while the implied volatility surface changes over time, its shape tends to pervade. This motivates us to construct a dynamic model for implied volatility surface, which not only captures cross-sectional information of implied volatilities with different strikes and maturities, but also describes how the implied volatility surface evolves over time. In this paper, we use nonlinear parametric function to capture single implied volatility surface, and model the dynamics of implied volatility surface by modeling the dynamics of function coefficients. We introduce unscented Kalman filter to propagate the nonlinear system, which is constructed by the nonlinear parametric function and the dynamics of its coefficients. A dynamic approach is proposed to provide optimal estimation of model parameters and efficient forecast of future implied volatility surface. It shows that our model has a better description of implied volatility surface dynamics than other similar models, and can be used to do volatility surface forecast.

1. Introduction

The inconsistency in Black-Scholes model is widely known, as implied volatilities are different across strikes and time to maturities. For a given time to maturity, implied volatilities often exhibit a smile/skew pattern across strikes. When considering different expirations, volatility skews extend to a non-flat volatility surface. Handling the implied volatility surface appropriately is critical to trading desks. From the perspective of market making for exchange-traded options, the pricing errors need to be small and preferably within the bid-ask spread, as a market maker can rarely take views against the whole market. From the perspective of pricing and hedging exotic options, the model should fit the market implied volatilities accurately to ensure all derivatives are priced under a consistent system.

The obvious shortcomings of the Black-Scholes model have triggered a large number of literatures, in which both academics and practitioners tried to account for its limitations. One strand resorted to local volatility models, (Derman and Kani, 1994). This model can be calibrated to fit the current implied volatility surface nearly perfectly, but the same model parameters might produce a poor fit to the surface in the short future. As a result, frequent re-calibration is necessary, which forms a highly variant time dynamics of local volatility parameters. Theoretically, this time instability of model parameters suggests its incompetence to capture the kernel of the surface dynamics. Practically, it leads to large variations of hedging parameters, and hence is problematic for risk management applications, (Cont et al., 2002).

Another strand resorted to stochastic volatility and jump models. For stochastic volatility models, the dynamic of underlying is specified under risk-neutral measure, and additional risk sources are embedded in the spot volatility process, see Heston (1993) and Hagan et al. (2002) for example. Besides, various Lévy jump processes have been proposed to capture smiles at short maturities, see Bates (1996); Kou (2002) and Zhang et al. (2012). Besides, Jumps and stochastic volatilities have been combined in various forms to capture implied volatility behaviors over different strikes, maturities, and calendar days (Bakshi et al., 1997; Bakshi et al., 2008; Carr and Wu, 2004). These models have five disadvantages. First, they model unobservable spot volatility of underlying asset while the market usually quotes options directly in terms of their implied volatilities. So, the volatility surface cannot be represented by their model parameters directly. Second, although it is possible to get implied volatility surface numerically, almost all these stochastic volatility/jump models yield a smooth, but usually imperfect, fitting of the observed implied volatility surface. Third, the problem just stated at the static level, becomes more acute at dynamic level. Fourth, model dynamics implied from option price data and extracted from

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underlying time series can be different. Fifth, it is very hard to estimate the parameters associated with jumps. These shortcomings raise the need for a more robust model of the surface dynamics.

In short, stochastic volatility model may not be able to reproduce the surface, and local volatility model tends to overfit the surface and exhibit undesirable temporal variation. All of these indicate that option prices can be affected by factors other than the underlying asset. In order to accurately capture the volatility dynamics, both underlying and option prices are necessary, (Li, 2013). There is a need to use information from both underlying market and option market, and raise a model which can describe the evolution of volatility surface well.

Other scholars and practitioners chose to model the observable implied volatility surface directly, instead of modeling the unobservable spot volatility. Existing works of dynamic implied volatility models can be classified into three branches. The first branch is $market-based$ models, where joint dynamics for underlying asset and implied volatilities are specified, see Schönbucher (1999) and Ledoit and Santa-Clara (1998). They specify the continuous martingale component of the implied volatility surface, and take the observed implied volatility as given, in order to derive no-arbitrage drift restriction on the surface. A similar approach led to the well-known Heath-Jarrow-Morton drift conditions. A common criticism against this approach is that the knowledge of initial implied volatility surface places unclear constraints on the specification of the continuous martingale component of its subsequent dynamics, (Carr and Wu, 2013). The second branch is proposed by Carr and Wu (2010, 2013), named the $Vega-Gamma-Vanna-Volga$ model. They only specify the implied volatility dynamics, of which the drift part and diffusion part are represented by several time-varying parameters, and leave the spot volatility process unspecified. The shape of the implied volatility surface is governed by a fundamental partial differential equation, which is derived under the condition that the discounted prices of options and their underlying are martingales under the risk-neutral measure. For this approach, improper specification of implied volatility dynamics would end up in unrealistic shape of model volatility surface. The third branch is $factor-based$ models, which assumes the implied volatility surface can be described as a randomly fluctuating surface driven by a small number of factors, as pointed out by large number of literatures. Cont & Fonseca (2002), Fengler et al. (2003) and Skiadopoulos et al. (2000) use non-parametric method, such as Karhunen-Loéve decomposition, common principal components analysis and principal components analysis respectively, to extract these factors and study their dynamics. Nonparametric methods tend to be very data intensive, and factors obtained by this way only have statistical meanings, and hence constrain the further use of such models. Other authors, including Hanfer (2005) and Goncalves et al. (2006) use linear parametric functions to describe the shape of implied volatility surface across different strikes and time to maturities, and propose a two-stage approach to estimate the dynamic properties of volatility surface by exploiting both cross-sectional and time series information. The problem is, error in the first step may cause remarkable inaccuracy in the second step. So Bedendo and Hodges (2009) adopts one-step Kalman filter approach to provide a more accurate and robust estimation for the model parameters, and it is the first application of Kalman filter to the updating of volatility skew.

Following the factor-based approach, we suggest a general model for the dynamics of the implied volatility surface in this paper. Compared to Bedendo and Hodges (2009), we don't restrict the measurement equation–which is used to capture single implied volatility surface at each time–to be linear, since many non-linear parameterizations have much better fitting accuracy and theoretical backgrounds. Coefficients of the measurement equation are factors driving the evolution of the whole surface, whose dynamics can be described by a set of stochastic differential equations. The measurement function together with the dynamics of its coefficients constructs a nonlinear system. Kalman Filtering techniques are famous for its ability to estimate dynamics systems. Since linear Kalman filter is not suitable for nonlinear system, unscented Kalman filter is introduced to deal with the nonlinearity. Unlike the Vega-Gamma-Vanna-Volga model, which first specifies the dynamic process of the implied volatility, and then derives a function representing single implied volatility surface, we first choose a specific parametric function for single implied volatility surface, and then use Itó's Lemma to derive the dynamics of the implied volatility. This difference frees our model from the problem in Vega-Gamma-Vanna-Volga model that the inappropriate specification of the dynamics of the implied volatility could bring about an unrealistic volatility surface. Compared to Junye Li (2013), which only uses at-the-money option prices for the simulation of underlying and spot volatility dynamics, in this paper, data of the whole surface are taken into consideration. After generating implied volatilities, Black-Scholes formula can be used to calculate vanilla option prices and exposures to risk factors, such as *vanna* and *volga*. It provides a framework under which both standard exchange-traded options and over-the counter exotic options can be priced and hedged consistently. Besides, our model provides an efficient way to estimate and forecast the volatility surface as a whole, while most articles on volatility forecast only focus on historical volatility or implied volatility of at-the-money options.

In what follows, a widely-used SABR parametrization (Hagan et al., 2002) is chosen as an example to investigate the application of our work in practice. Comparisons are drawn in terms of in-sample fitting accuracy and out-of-sample forecast accuracy, between linear parameterizations and nonlinear parameterizations from several aspects, to justify the use of nonlinear parameterizations. Moreover, as most of the existing works are based on daily data, our empirical study is based on intra-day minute data of S&P 500 index options, which makes it meaningful for practitioners, especially exchange-traded option market makers, since most of their work involves the surface variation during the day. According to empirical study, our model can generate a stable fit to volatility surfaces, while dynamic implied volatility model with linear measurement equation can generate large errors at some time points during the propagation process. It shows that our model has a better description of implied volatility surface dynamics than other similar models, and can be used to do volatility surface forecast.

This paper is structured as follows. In Section 2, we first review the knowledge of implied volatility surface. Section 2.2 displays our model set-up. Section 2.3 introduces the unscented Kalman filter and its implementation. Model comparison and data analysis are showed in Section 3. Section 3.2 introduces the famous SABR parameterization and extends the original SABR model to capture the evolution of whole volatility surface, instead of volatility skew for a particular expiration. Results of parameter estimation are listed in Section 3.3. In Section 3.4, nonlinear parameterizations and linear parameterizations are compared from several aspects: in-sample fitting accuracy, out-of-sample forecast performance, the ability to fit the whole volatility surface, errors of forecast and posterior estimation for dynamic implied volatility model. Section 4 is the conclusion.

2. The stochastic implied volatility model based on parametric factors

2.1. The implied volatility surface

The implied volatility $\hat{\sigma}$ of an option is defined to be the value of the volatility parameters for the Black-Scholes formula that makes the Black-Scholes price equal to the option market price. The Black-Scholes formula for a call option on a non-dividend paying asset S with expiration T, and strike price K, at time t, is

$$C(S,t,K,T,r,\sigma) = SN(d_1) - e^{-r(T-t)}KN(d_2) \quad (1)$$

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